

Ecuaciones de un sumador completo (*full adder*)

$$s_i = a_i b_i' c_i' + a_i' b_i c_i' + a_i' b_i' c_i + a_i b_i c_i$$

$$c_{i+1} = a_i b_i + a_i c_i + b_i c_i$$

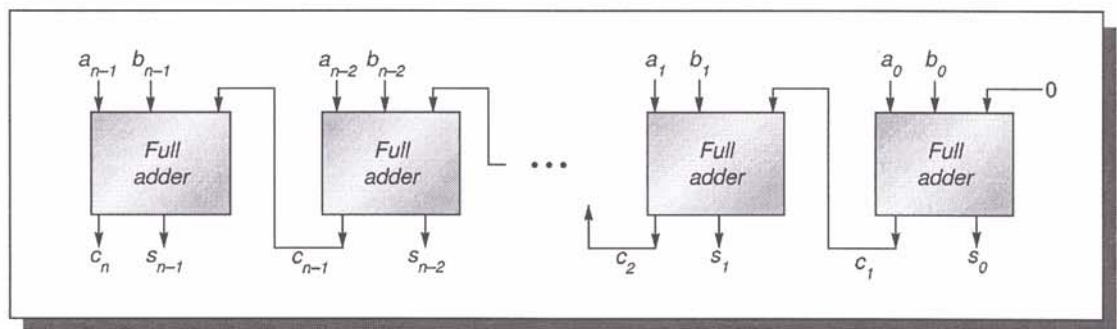


FIGURE A.1 Ripple-carry adder, consisting of n full adders.

$$c_{i+1} = a_i b_i + a_i c_i + b_i c_i$$

$$c_{i+1} = g_i + p_i c_i$$

$$g_i = a_i b_i$$

$$p_i = a_i + b_i$$

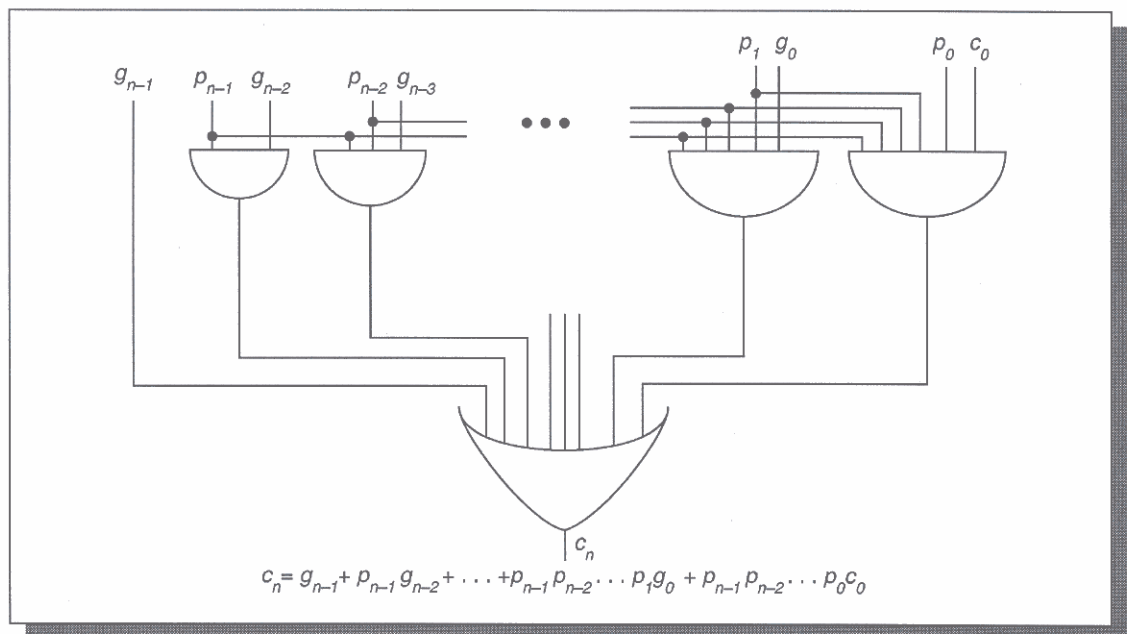


FIGURE A.14 Pure carry-lookahead circuit for computing the carry-out c_n of an n -bit adder.

$$c_{j+1} = G_{ij} + P_{ij}c_i$$

$$G_{ik} = G_{j+1,k} + P_{j+1,k}G_{ij}$$

$$P_{ik} = P_{ij}P_{j+1,k}$$

$$P_{ii} = p_i; G_{ii} = g_i$$

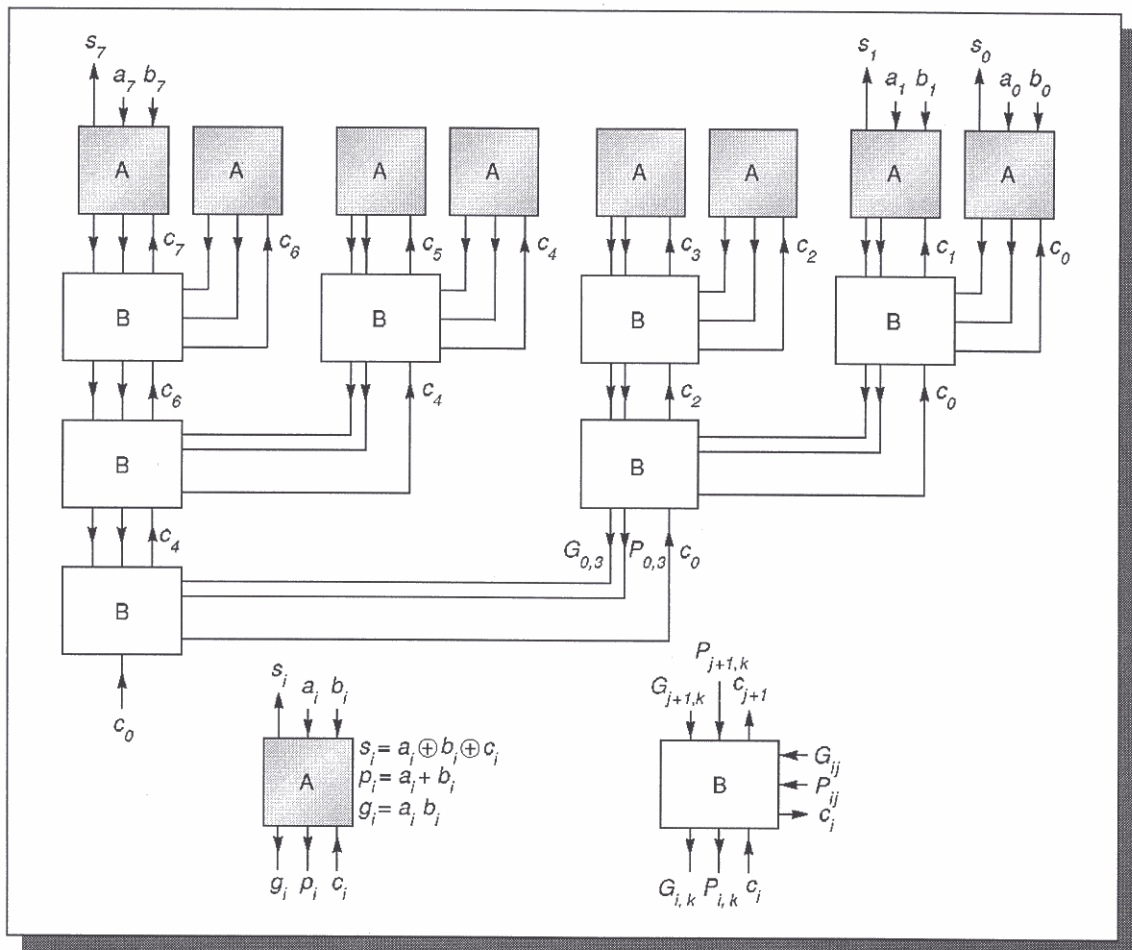


FIGURE A.17 Complete carry-lookahead tree adder.

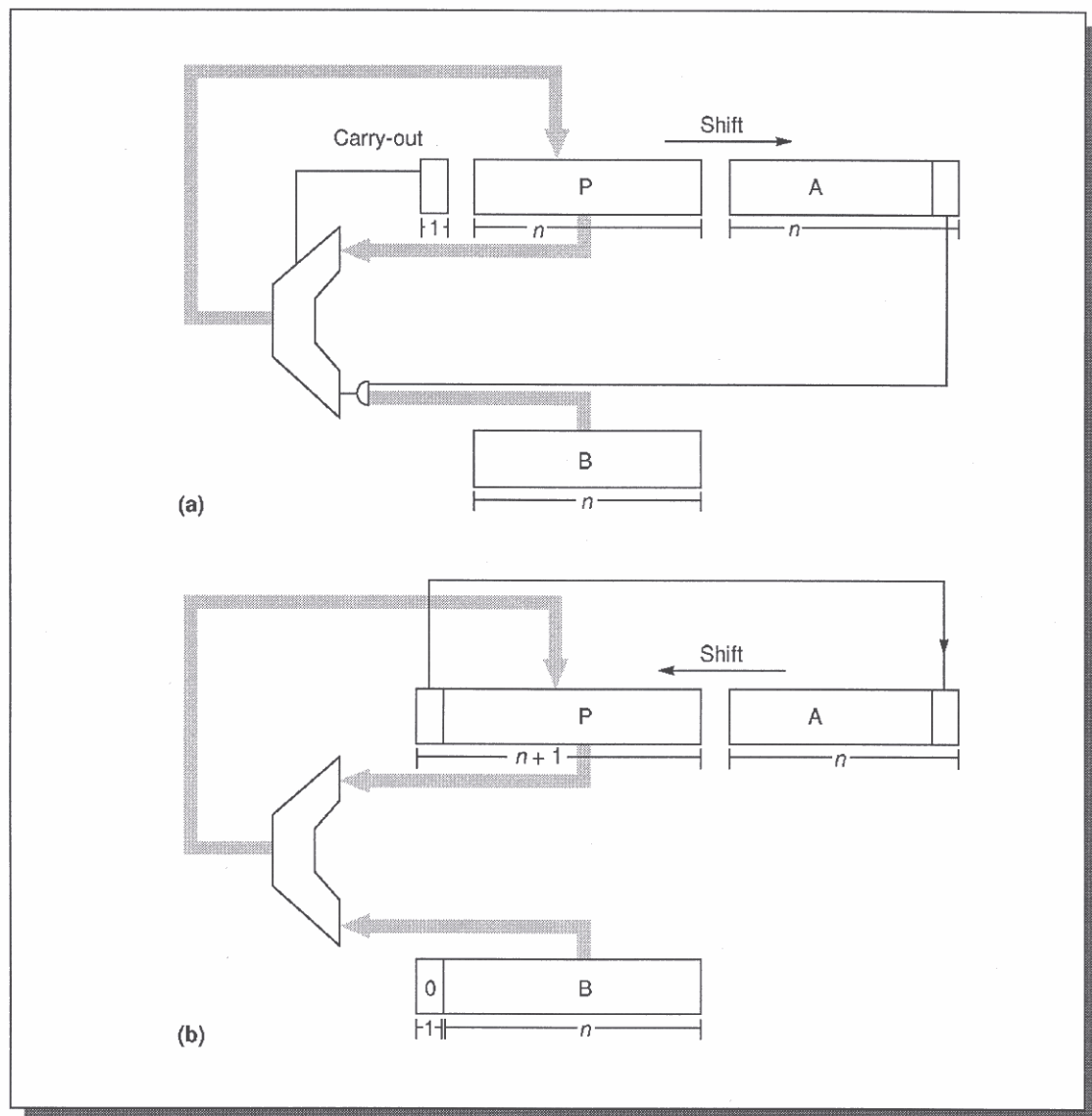


FIGURE A.2 Block diagram of (a) multiplier and (b) divider for n -bit unsigned integers.

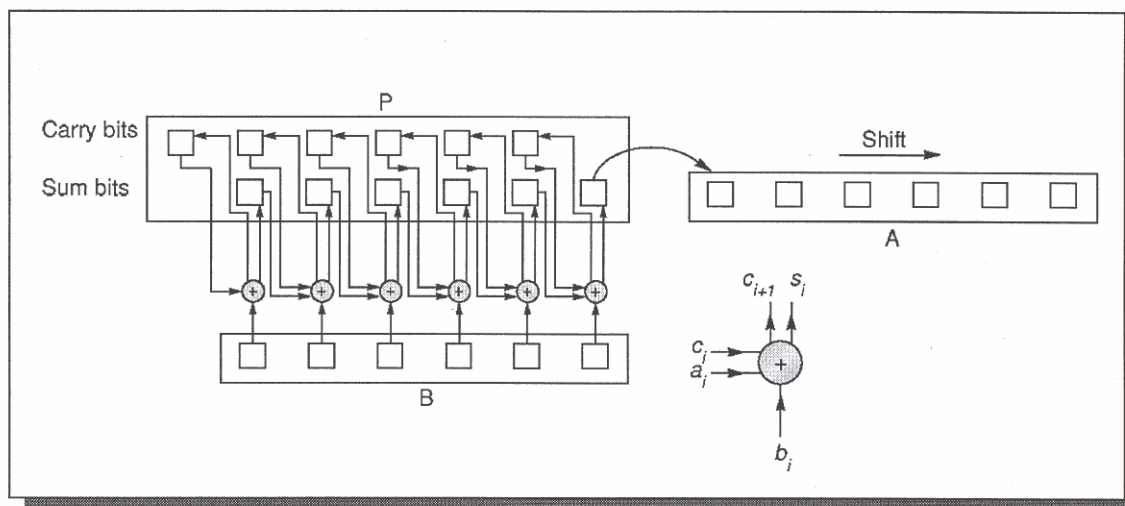


FIGURE A.24 Carry-save multiplier.

Low-order bits of A		Last bit shifted out		Multiple
$2i + 1$	$2i$	$2i - 1$		
0	0	0		0
0	0	1		$+b$
0	1	0		$+b$
0	1	1		$+2b$
1	0	0		$-2b$
1	0	1		$-b$
1	1	0		$-b$
1	1	1		0

FIGURE A.25 Multiples of b to use for radix-4 Booth recoding.

P	A	L	
00000	1001		Multiply $-7 = 1001$ times $-5 = 1011$. B contains 1011.
<u>+ 11011</u>			Low order bits of A are 0, 1; L=0, so add B.
11011	1001		
11110	1110	0	Shift right by two bits, shifting in 1s on the left.
<u>+ 01010</u>			Low order bits of A are 1, 0; L=0, so add $-2b$.
01000	1110	0	
00010	0011	1	Shift right by two bits.
			Product is $35 = 0100011$.

FIGURE A.26 Multiplication of -7 times -5 using radix-4 Booth recoding.

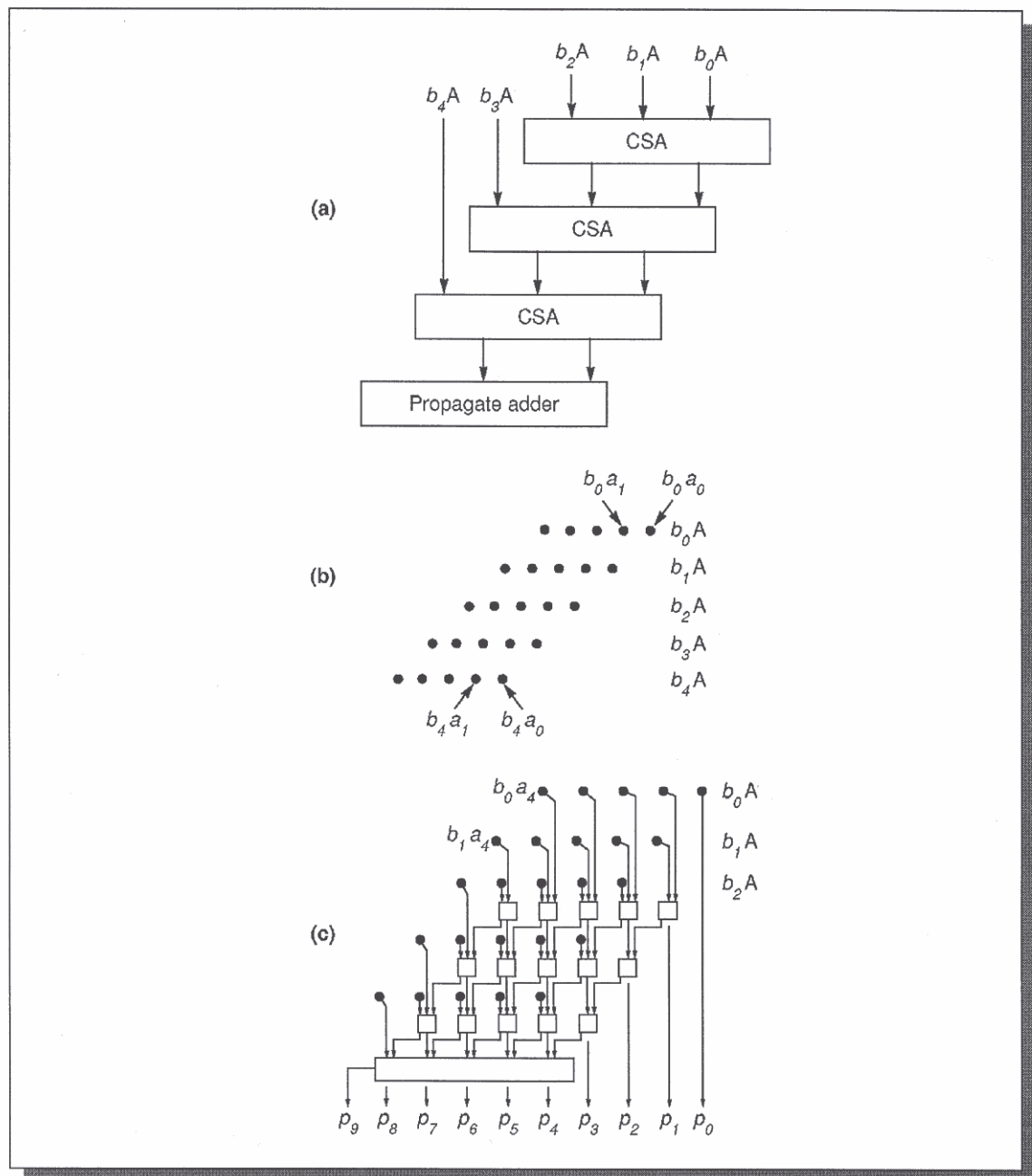


FIGURE A.27 An array multiplier.

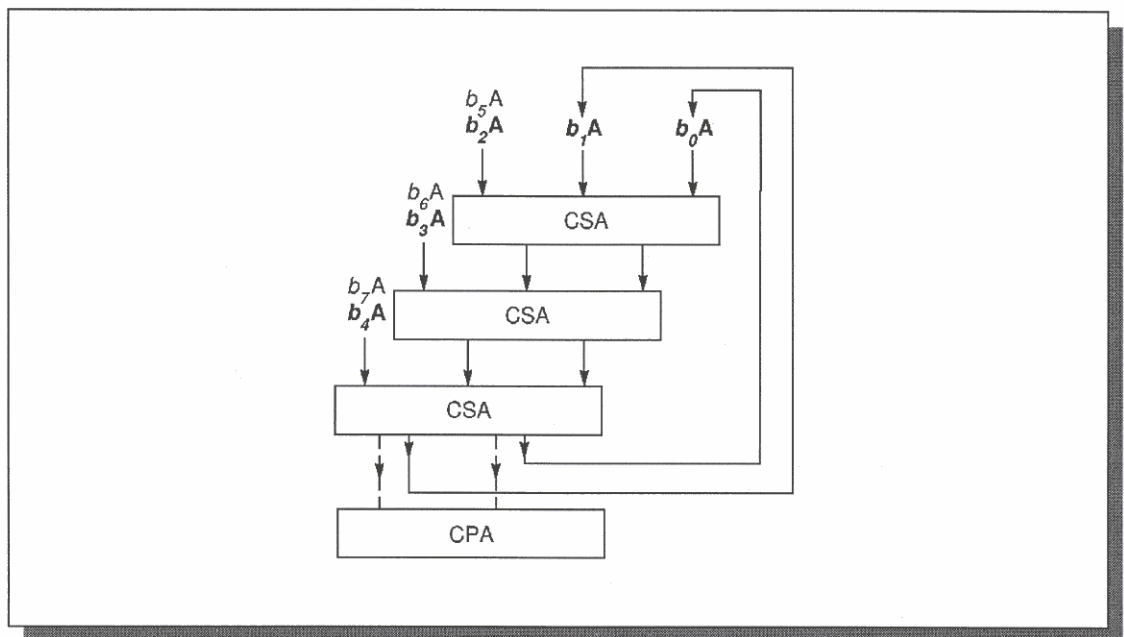


FIGURE A.28 Multipass array multiplier.

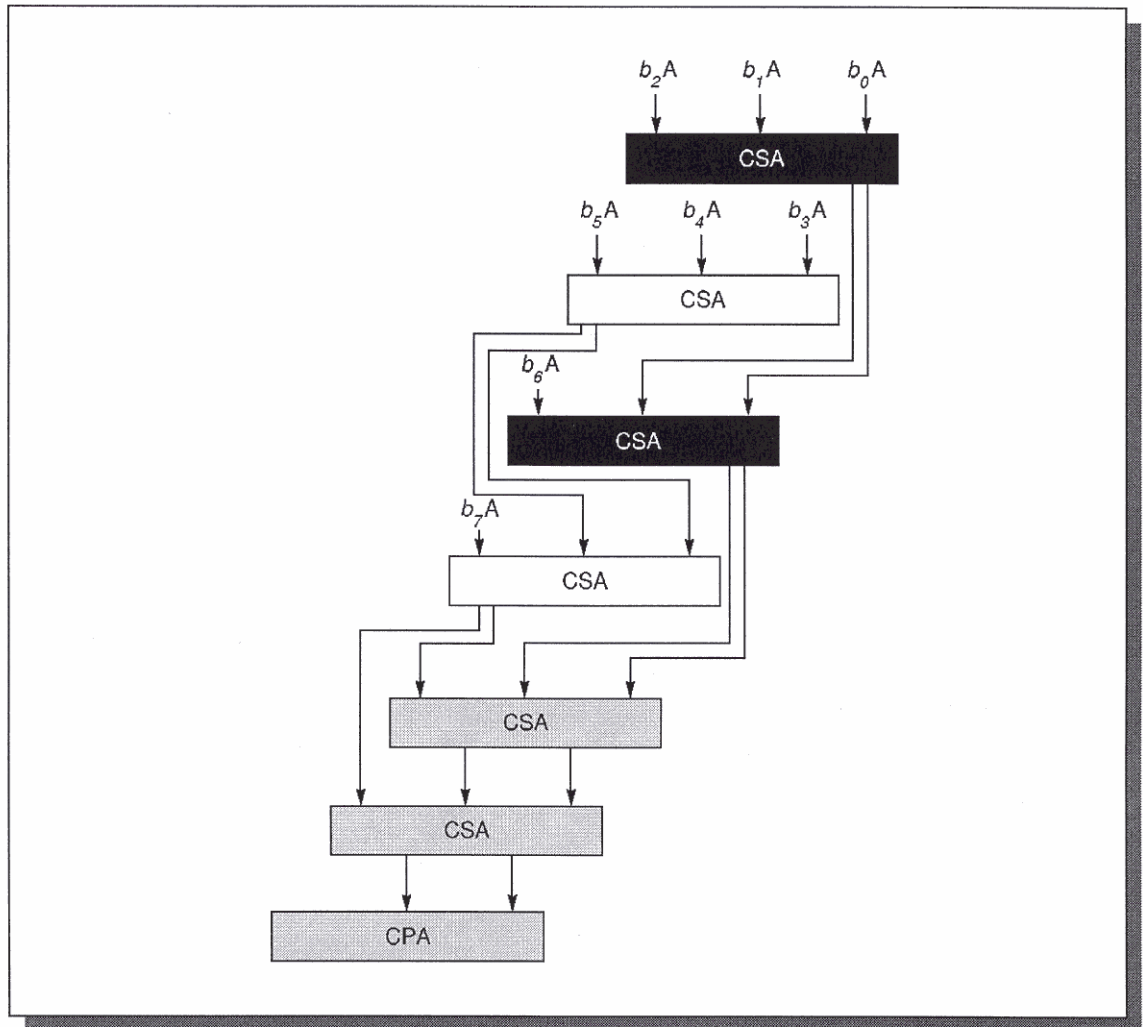


FIGURE A.29 Even/odd array.

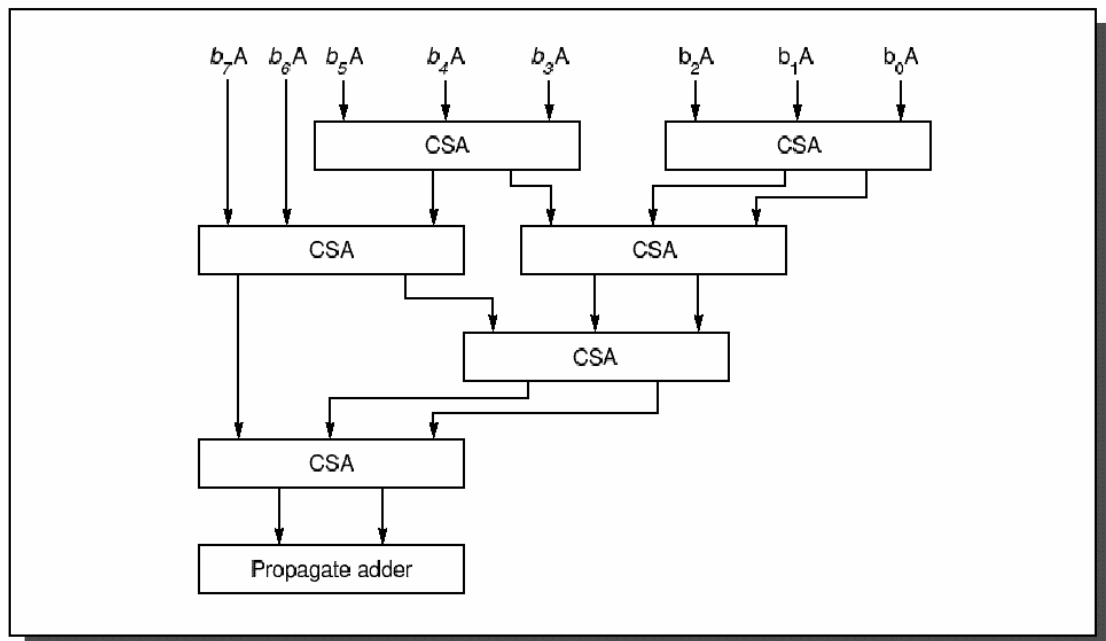


FIGURE A.30 Wallace tree multiplier.

$\begin{array}{r} 1 \\ +1 \\ \hline 10 \end{array}$	$\begin{array}{r} 1 \\ +\bar{1} \\ \hline 00 \end{array}$	$\begin{array}{r} \bar{1} \\ +\bar{1} \\ \hline \bar{1}0 \end{array}$	$\begin{array}{r} 0 \\ +0 \\ \hline 00 \end{array}$	$\begin{array}{r} 1 \ x \\ +0 \ y \\ \hline 1 \ \bar{1} \end{array}$ if $x \geq 0$ and $y \geq 0$ 0 1 otherwise	$\begin{array}{r} \bar{1} \ x \\ +0 \ y \\ \hline 0 \ \bar{1} \end{array}$ if $x \geq 0$ and $y \geq 0$ $\bar{1} \ 1$ otherwise
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FIGURE A.31 Signed-digit addition table.

DIVISIÓN SRT

/* División de a entre b (ambos de n bits). Ruta de datos: ver transparencia *Mult/div enteras-1* */

A := a; B := b; P := 0;

if “B, expresado con n bits, tiene k ceros a la izda.” then “desplazar B y (P&A) k bits a la izquierda”; /* A1 */

for i:=0 to n-1 do

{ a: if “los tres bits a la izda. de P son iguales” then /* A2 */

{“desplazar (P&A) un bit a la izquierda”; $q_i := 0$ } /* A3 */

else

b: if $P < 0$ then /* A4 */

{“desplazar (P&A) un bit a la izquierda”; $q_i := -1$; “sumar B”};

/* A5 */

else /* A6 */

c: {“desplazar (P&A) un bit a la izquierda”; $q_i := 1$; “restar B”};

/* A7 */

};

if $P < 0$ then {“sumar B”; restar 1 al cociente q };

“desplazar el resto (P), k bits a la derecha”

P	A	
00000	1000	Divide $8 = 1000$ by $3 = 0011$. B contains 0011.
00010	0000	step 1: B had two leading 0s, so shift left by 2. B now contains 1100.
00100	0000	step 2.1: Top three bits are equal. This is case (a), so set $q_0 = 0$ and shift.
01000	0001	step 2.2: Top three bits not equal and $P \geq 0$ is case (c), so set $q_1 = 1$ and shift. Subtract B.
+ 10100	0001	step 2.3: Top bits equal is case (a), so
11100	0010	set $q_2 = 0$ and shift.
11000	0101	step 2.4: Top three bits unequal is case (b), so set $q_3 = -1$ and shift. Add B.
+ 01100		step 3. Remainder is negative so restore it and subtract 1 from q .
11100		
+ 01100		
01000		Must undo the shift in step 1, so right shift by 2 to get true remainder. Remainder = 10, quotient = $010\bar{1} - 1 = 0010$.

FIGURE A.23 SRT division of $1000_2/0011_2$.

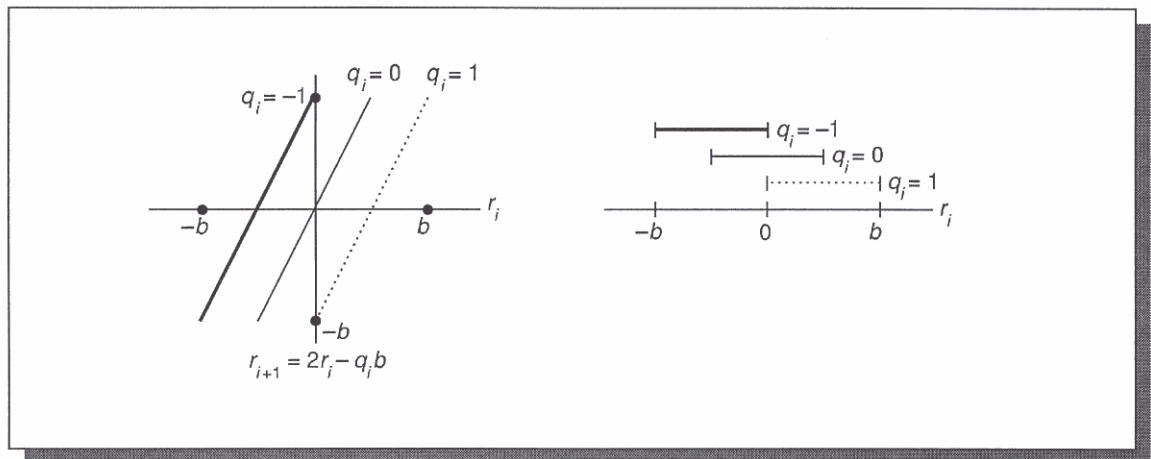


FIGURE A.32 Quotient selection for radix-2 division.

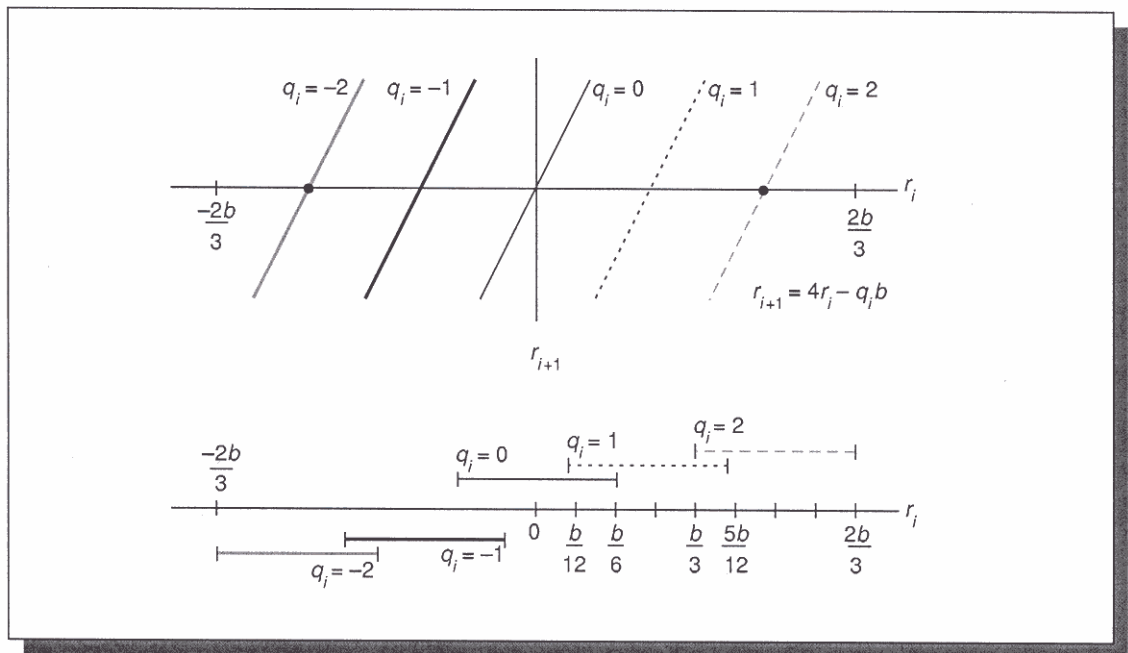


FIGURE A.33 Quotient selection for radix-4 division with quotient digits $-2, -1, 0, 1, 2$.

b	Range of P		q	b	Range of P		q
8	-12	-7	-2	12	-18	-10	-2
8	-6	-3	-1	12	-10	-4	-1
8	-2	1	0	12	-4	3	0
8	2	5	1	12	3	9	1
8	6	11	2	12	9	17	2
9	-14	-8	-2	13	-19	-11	-2
9	-7	-3	-1	13	-10	-4	-1
9	-3	2	0	13	-4	3	0
9	2	6	1	13	3	9	1
9	7	13	2	13	10	18	2
10	-15	-9	-2	14	-20	-11	-2
10	-8	-3	-1	14	-11	-4	-1
10	-3	2	0	14	-4	3	0
10	2	7	1	14	3	10	1
10	8	14	2	14	10	19	2
11	-16	-9	-2	15	-22	-12	-2
11	-9	-3	-1	15	-12	-4	-1
11	-3	2	0	15	-5	4	0
11	2	8	1	15	3	11	1
11	8	15	2	15	11	21	2

FIGURE A.34 Quotient digits for radix-4 SRT division with a propagate adder.

	Single	Single extended	Double	Double extended
p (bits of precision)	24	≥ 32	53	≥ 64
$E_{\max}$	127	≥ 1023	1023	≥ 16383
$E_{\min}$	-126	≤ -1022	-1022	≤ -16382
Exponent bias	127		1023	

FIGURE A.7 Format parameters for the IEEE 754 floating-point standard.

Exponent	Fraction	Represents
$e = E_{\min} - 1$	$f = 0$	± 0
$e = E_{\min} - 1$	$f \neq 0$	$0.f \times 2^{E_{\min}}$
$E_{\min} \leq e \leq E_{\max}$	—	$1.f \times 2^e$
$e = E_{\max} + 1$	$f = 0$	$\pm \infty$
$e = E_{\max} + 1$	$f \neq 0$	NaN

FIGURE A.8 Representation of special values.

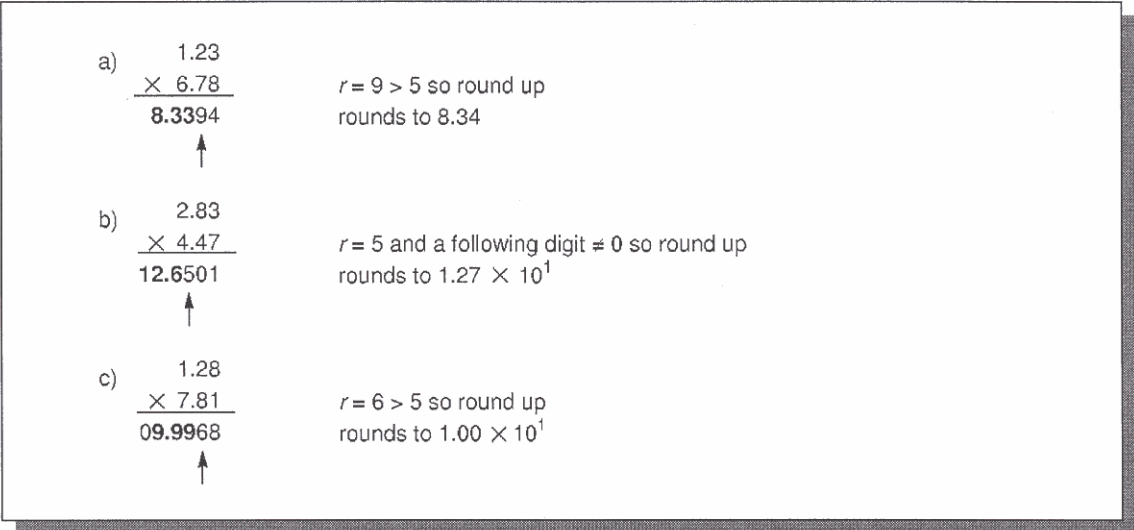


FIGURE A.9 Examples of rounding a multiplication.

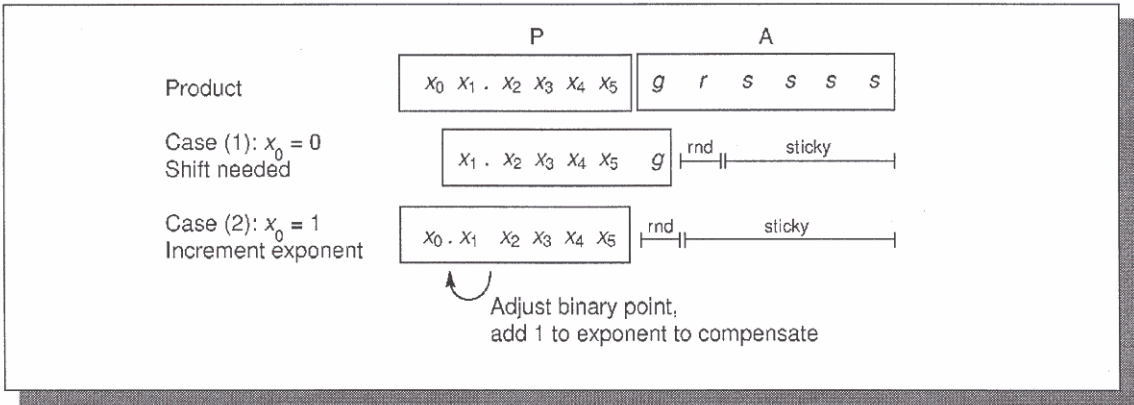


FIGURE A.10 The two cases of the floating-point multiply algorithm.

Rounding mode	Sign of result ≥ 0	Sign of result < 0
$-\infty$		+1 if $r \vee s$
$+\infty$	+1 if $r \vee s$	
0		
Nearest	+1 if $r \wedge p_0$ or $r \wedge s$	+1 if $r \wedge p_0$ or $r \wedge s$

FIGURE A.11 Rules for implementing the IEEE rounding modes.

Ejemplos de sumas/restas en punto flotante con 6 bits de precisión (mantisas en base 2).

1º) $1,10011 + 1,10001 \times 2^{-5}$

$$\begin{array}{r} 1,10011 \\ + \underline{,0000110001} \end{array}$$

Con un sumador de 6 bits

$$\begin{array}{r} 1,10011 \\ + \underline{,00001} \\ 1,10100 \end{array}$$

Bit de redondeo (en negrita) = 1. Bit retenedor $s = 1$ ($0 \vee 0 \vee 0 \vee 1$).

Redondeo (sumar 1) $\Rightarrow$ mantisa final = 1,10101

2º) $1,11011 + 1,01001 \times 2^{-2}$

$$\begin{array}{r} 1,11011 \\ + \underline{,0101001} \end{array}$$

Con un sumador de 6 bits

$$\begin{array}{r} 1,11011 \\ + \underline{,01010} \\ 10,00101 \end{array}$$

Bit de redondeo (en negrita) = 1. Bit retenedor $s = 1$ ($0 \vee 1$).

Redondeo (sumar 1) $\Rightarrow$ mantisa final = 10,0011

3º) $1,00000 - 1,01111 \times 2^{-6}$

$$\begin{array}{r} 1,00000 \\ - \underline{,00000101111} \end{array}$$

Usamos el complemento a 2 del sustraendo para convertir la resta en suma

$$\begin{array}{r} 1,00000 \\ + \underline{1,11111010001} \end{array}$$

Con un sumador de 6 bits

$$\begin{array}{r} 1,00000 \\ + \underline{1,11111} \\ 0,11111 \end{array}$$

Bit de guarda (negrita) = 0 $\Rightarrow$ Resultado sin redondeo = 0,111110.

Bit de redondeo (negrita) = 1. Bit retenedor $s = 1$ ($0 \vee 0 \vee 0 \vee 1$).

Redondeo (sumar 1) $\Rightarrow$ mantisa final = 0,111111.

ALGORITMO DE SUMA EN PUNTO FLOTANTE

/* a_1 y a_2 : números por sumar.

e_i : exponente de a_i . s_i : mantisa desempaquetada de a_i */

- 1: **if** $e_1 < e_2$ **then** {“intercambiar a_1 y a_2 ”};
 $d := e_1 - e_2$ /* $d \geq 0$ */; $e := e_1$ /* exponente inicial del resultado */
- 2: **if** $\text{sign}(a_1) \neq \text{sign}(a_2)$ **then** $s_2 := \text{Ca}_2(s_2)$ /* Ca_2 : complemento a 2 */
- 3: /* PS2: registro de p bits */ $\text{PS}_2 := s_2$;
 “desplazar PS_2 , d posiciones a la derecha
 (entrando unos por la izda. si s_2 se complementó en el paso 2) y,
 de los bits que han salido por la derecha de PS_2 , llamar g al más
 significativo, r al siguiente y s a la disyunción lógica de los demás”
- 4: $S := s_1 + \text{PS}_2$;
 if $\text{sign}(a_1) \neq \text{sign}(a_2) \wedge (\text{Bit más significativo de } S) = 1 \wedge$
 $\wedge$ no hay acarreo de salida
 then /* $S < 0$ ($d=0$) */ $S := \text{Ca}_2(S)$
- 5: **if** $\text{sign}(a_1) = \text{sign}(a_2) \wedge$ hay acarreo de salida
 then “desplazar S una posición a la dcha. e introducir un 1 (el acarreo) por la
 izda.”
 else “desplazar S hacia la izda hasta que quede normalizado (en el primer
 desplazamiento introducir por la derecha el bit g , en los siguientes introducir
 ceros)”;
 “Ajustar el exponente del resultado, e ”
- 6: **if** “ S fue desplazado a la derecha” **then**
 { $s := g \vee r \vee s$; $r :=$ “bit menos significativo de S antes de desplazar” };
 if “ S no fue desplazado” **then** { $s := r \vee s$; $r := g$ };
 if “ S fue desplazado 2 ó más posiciones a la izquierda”
 then { $r := 0$; $s := 0$ }
- 7: “redondear S ”; **if** “se produce acarreo de salida” **then**
 “desplazar S un bit a la dcha., introducir un 1 por la izda. y ajustar el exponente, e ”
 /* S es la mantisa y e el exponente del resultado */
- 8: /* Calcular el signo del resultado, sr */
 if $\text{sign}(a_1) \neq \text{sign}(a_2) \wedge e_1 = e_2$ **then**
 if “se complementó S en el paso 4” /* $|\text{PS}_2| > s_1$ */
 then $sr := \text{sign}(a_2)$ **else** $sr := \text{sign}(a_1)$
 else $sr := \text{sign}(a_1)$ /* $e_1 > e_2 \vee \text{sign}(a_1) = \text{sign}(a_2)$ */

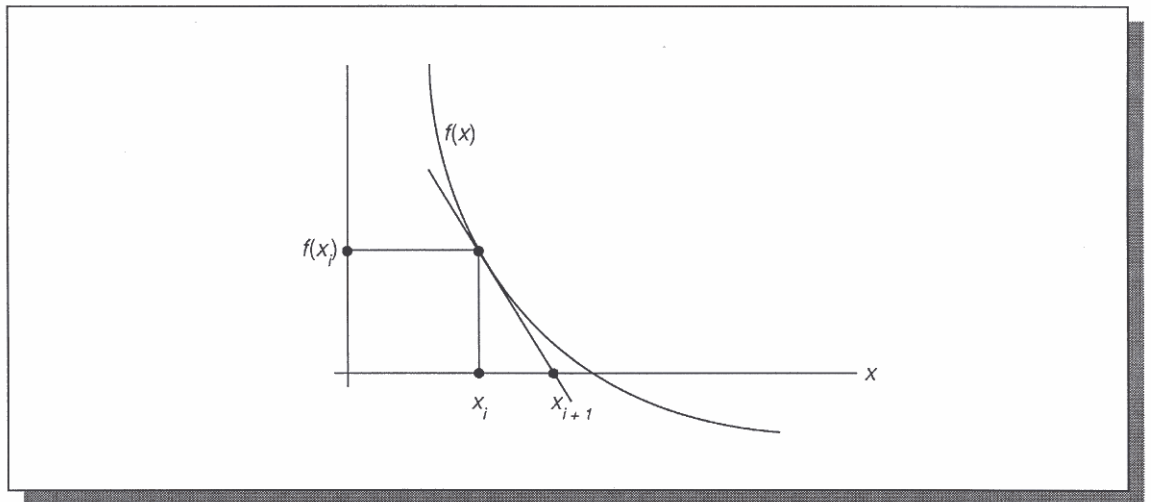


FIGURE A.13 Newton's iteration for zero finding.

Ecuación de la tangente a la curva:

$$y - f(x_i) = f'(x_i) (x - x_i)$$

Corte con el eje de abscisas ($y = 0$):

$$-f(x_i) = f'(x_i) (x - x_i)$$

$$x_{i+1} = x = x_i - f(x_i)/f'(x_i)$$

Aplicación a la función

$$f(x) = x^{-1} - b$$

Se tiene

$$f'(x) = -x^{-2}$$

y, por tanto,

$$x_{i+1} = x_i - f(x_i)/f'(x_i) = x_i + x_i - b x_i^2$$

En definitiva

$$x_{i+1} = 2 x_i - b x_i^2 = x_i (2 - b x_i)$$