MODELOS MATEMÁTICOS EN SISTEMAS DE TRANSPORTES
Escuela Superior de Informática Universidad de Castilla-La Mancha
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Aplicaciones del diseño de redes:
Diseño de servicios de metro y suburbano
Diseño de aeropuertos y de su área terminal

por

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Indice

- ✓ Rail Transportation Planning
- ✓ Network design models
- ✓ Service network design models
- ✓ Rapid Transit Network Design (RTND)
- ✓ Robustness RTND and capacitated network design models
- ✓ Taxi Planning: Routing and scheduling models
- ✓ Airport Terminal Area Design

Active Transportation Projects

- ***Project:*** ***"Aplicaciones del diseño de redes de transporte".***

  Ministerio de Educación y Ciencia, 2006 to 2008. Since 2000.

- **Project:** ***"Optimización Matemática para la planificación robusta y la extensión estratégica de sistemas metropolitanos de transporte público".***

  Ministerio de Fomento, 2006 and 2007. Since 2004.

- **Project: ARRIVAL** ***"Algorithms for Robust and on-line Railway optimization: Improving the Validity and reliability of Large-scale systems".***

  European Commission. Sixth Frame Program, 2006 to 2008.

Rail Transportation Planning

Long term planning: Strategic railway planning (Infrastructure problem)

- Uncapacitated facility location (stations and alignments location)
- Uncertainty demand.
- The frequency of the lines is a data.

Medium term planning: Line planning (Fleet problem)

- Capacitated facility location (train lines).
- To satisfy a known and deterministic traffic.
- The frequency of operations is a variable.

Short term planning: Timetable (Routing and scheduling problem)

- Concrete use of a given capacity (timetabling, resource scheduling: assigns locomotives, cars, and crew to the rides).
- Dynamic demand. Space-temporal networks.
- The timetable is a variable and the optimal frequency is known.

Rail Transportation Planning

Long term planning: Strategic railway planning (Infrastructure problem)

- Network design (stations and alignments location)
- Network design with uncertainty demand.
- Capacity expansion.

Medium term planning: Line planning (Fleet problem)

- Service (line) planning

Short term planning: Timetable (Scheduling problem)

- Timetable , resource scheduling: assigning locomotives

Robustness

- Flow and time reliability constraints
- Between strategic and tactical planning
- New concepts of robustness

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Multicommodity uncapacitated Network

$$\underset{x \in R^+}{Min.} \sum_{w \in W} \sum_{ij \in A} c_{ij}^w x_{ij}^w$$

$$\sum_{j:ji \in A} x_{ji}^w - \sum_{j:ij \in A} x_{ij}^w = b_i^w = \begin{cases} 1, if\ i \in d(w) \\ -1, if\ i \in o(w) \\ 0, otherwise. \end{cases}, \forall i, \forall w$$

$$x_{ij}^w \leq 1, \forall ij, \forall w$$

Multicommodity capacitated Network

$$\underset{x \in R^{+}}{Min.} \sum_{w \in W} \sum_{ij \in A} c_{ij}^{w} x_{ij}^{w}$$

$$\sum_{j:ji \in A} x_{ji}^{w} - \sum_{j:ij \in A} x_{ij}^{w} = b_{i}^{w} = \begin{cases} g_{w}, if\ i \in d(w) \\ -g_{w}, if\ i \in o(w) \\ 0, otherwise. \end{cases}, \forall i, \forall w$$

$$\sum_{w \in W} x_{ij}^{w} \leq q_{ij}, \forall ij$$

Uncapacited Network Design

$$\underset{\substack{y\in\{0,1\}\\ x\in R^+}}{Min.} \sum_{w\in W}\sum_{ij\in A} c_{ij}^{w} x_{ij}^{w} + \sum_{ij\in A} f_{ij}\, y_{ij}$$

$$\sum_{j:ji\in A} x_{ji}^{w} - \sum_{j:ij\in A} x_{ij}^{w} = b_i^{w} = \begin{cases} 1, if\ i\in d(w) \\ -1, if\ i\in o(w) \ , \forall i, \forall w \\ 0, otherwise. \end{cases}$$

$$x_{ij}^{w} + x_{ji}^{w} \le y_{ij}, \forall ij\in A, i<j, \forall w$$

Capacitated Network Design

$$\underset{\substack{y\in\{0,1\}\\ x\geq 0}}{Min.} \sum_{w\in W}\sum_{ij\in A} c_{ij}^{w} x_{ij}^{w} + \sum_{ij\in A} f_{ij}\, y_{ij}$$

$$\sum_{j:\, ji\in A} x_{ji}^{w} - \sum_{j:ij\in A} x_{ij}^{w} = b_i^{w} = \begin{cases} g_w, if\ i\in d(w) \\ -g_w, if\ i\in o(w) \\ 0, otherwise. \end{cases}, \forall i, \forall w$$

$$\sum_{w\in W} x_{ij}^{w} \leq q_{ij} y_{ij}, \forall ij \in A$$

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Uncapacitated service network

$$\underset{\substack{y \in Z^{+} \\ x \in R^{+}}}{Min.} \sum_{w \in W} \sum_{ij \in A} c_{ij}^{w} x_{ij}^{w}$$

$$\sum_{r \in R_w} h_r = d_w = 1 \qquad \forall w \in W;$$

$$\sum_{r \in R_s} h_r \leq 1 \qquad \forall s \in S_l, \forall l \in L;$$

$$x_{ij}^{w} = \sum_{\substack{r \in R_w \\ r \in R_{ij}}} h_r, \forall ij \in A, \forall w \in W$$

Capacitated service network

$$\underset{\substack{y \in Z^+ \\ x \in R^+}}{Min.} \sum_{w \in W} \sum_{ij \in A} c_{ij}^{w} x_{ij}^{w}$$

$$\sum_{r \in R_w} h_r = d_w \qquad \forall w \in W;$$

$$\sum_{r \in R_s} h_r \leq q_s \qquad \forall s \in S_l, \forall l \in L;$$

$$x_{ij}^{w} = \sum_{\substack{r \in R_w \\ r \in R_{ij}}} h_r, \forall ij \in A, \forall w \in W$$

Uncapacitated service network design

$$\underset{\substack{y \in Z^+ \\ x \in R^+}}{Min} . \sum_{w \in W} \sum_{ij \in A} c_{ij}^{w} x_{ij}^{w} + \sum_{l \in L} f^{l} y_{l}$$

$$\sum_{r \in R_w} h_r = d_w = 1 \qquad \forall w \in W;$$

$$\sum_{r \in R_s} h_r \leq 1 \qquad \forall s \in S_l, \forall l \in L;$$

$$\sum_{l \in L} t_l y_l \leq fc$$

$$x_{ij}^{w} = \sum_{\substack{r \in R_w \\ r \in R_{ij}}} h_r, \forall ij \in A, \forall w \in W$$

Capacitated service network design

$$\underset{\substack{y \in Z^+ \\ x \in R^+}}{Min.} \sum_{w \in W} \sum_{ij \in A} c_{ij}^{w} x_{ij}^{w} + \sum_{l \in L} f^{l} y_{l}$$

$$\sum_{r \in R_w} h_r = d_w \qquad \forall w \in W;$$

$$\sum_{r \in R_s} h_r \leq q_s y_s \qquad \forall s \in S_l, \forall l \in L;$$

$$\sum_{l \in L} t_l y_l \leq fc$$

$$x_{ij}^{w} = \sum_{\substack{r \in R_w \\ r \in R_{ij}}} h_r, \forall ij \in A, \forall w \in W$$

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Rapid Transit Network Design

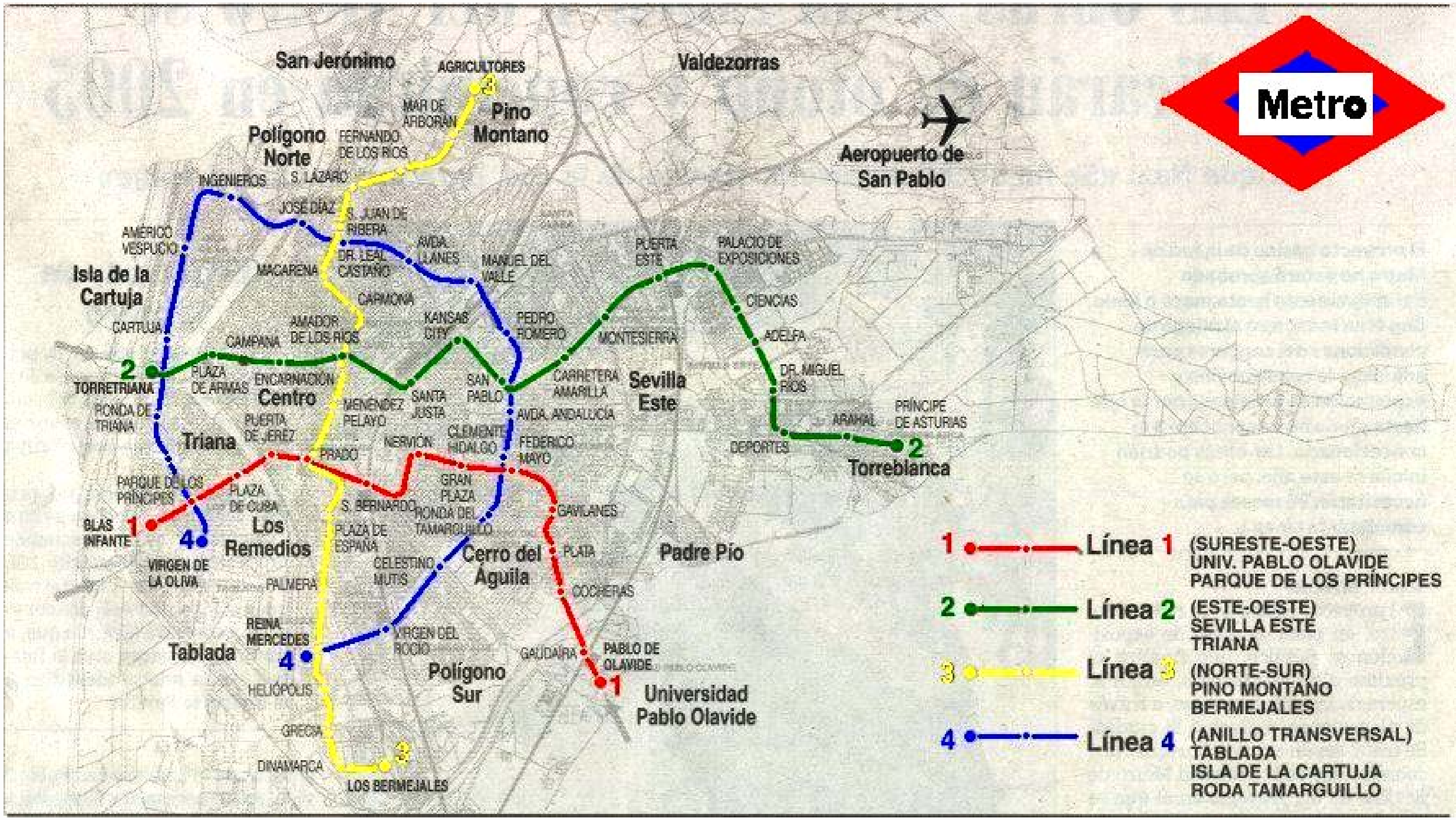
- **Higher level: Operators**

  1. **Objective: Maximize trip coverage by public mode**
  2. **Budget and line constraints**

- **Lower level: Users**

  1. **Users choose lower cost routes**
  2. **Users compare public and private costs**

Sevilla "metro" corridors



Rapid Transit Network Design
References

An integrated methodology (aligments+stations):
Laporte, Marín, Mesa, Ortega and Sevillano LNCS 2005
Designing networks in regard to transfers:
Garzón-Astolfi, Marín, Mesa and Ortega 2005
An extension to urban rapid transit network design:
Marín TOP 2006
A multi-modal approach to the location of the infraestructure of rapid transit network:
Marín and García 2006
Urban rapid transit network capacity expansion:
Marín y Jaramillo CLAIO'2006
Rapid Transit Network Design: Capacity Expansion
Marín and Jaramillo, accepted to EJOR 2007

Rapid Transit Network Design
Supply model

- **Public network design depends on the demand routing.**

- **Lines and stations must be located simultaneously.**

- **Lines are alignments of RTN, but RTN is a physical network no a service network. The line capacity is not considered (the train frequencies are parameters)**

Rapid Transit Network Design
Demand model

- **The demand is known and deterministic.**
- **The demand chooses the minimum cost routes (Second Wardrop Principle).**
- **The demand is mode share between public (PUB) and private (PRI).**

Rapid Transit Network Design
Objective Function

- **Maximize public demand coverage**

- **Minimize routing costs**

- **Minimize construction costs**

Rapid Transit Network Design Constraints

Location constraints:

The node and stations must be located making alignments without cycles.

Mode share constraints:

The demand is routed by PUB mode if the RTN (if it has been constructed) cost is inferior to the known cost by PRI mode.

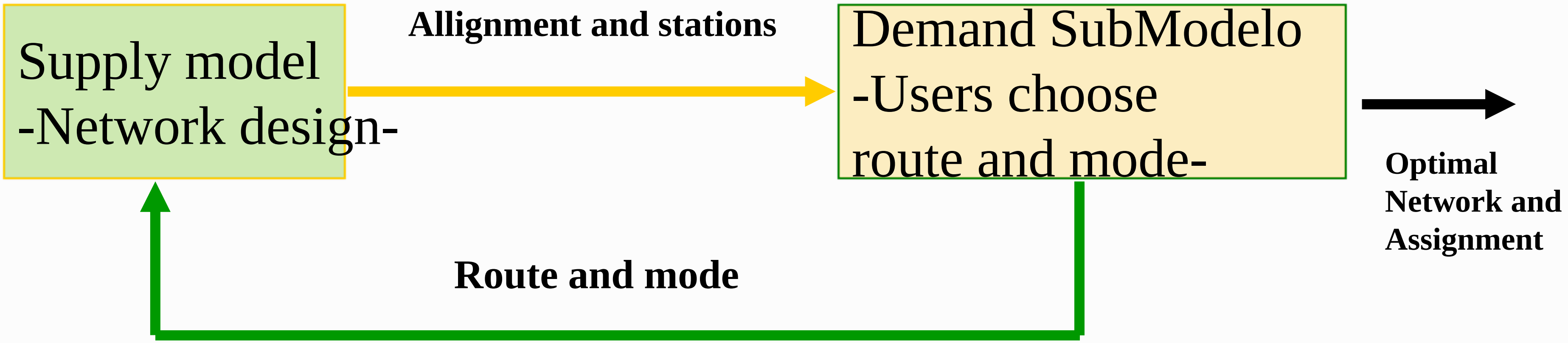
Routing constraints:

The demand is routed from origin to destination conserving the flow at nodes.

Location-allocation constraints

The public demand is routed only through the located RTN.

Rapid Transit Network Design



Rapid Transit Network: Uncapacited Network Design

$$\underset{\substack{y\in\{0,1\}\\ x\in R^+}}{Min.} \sum_{w\in W}\sum_{ij\in A} c_{ij}^{w} x_{ij}^{w} + \sum_{ij\in A} f_{ij}\, y_{ij}$$

$$\sum_{j:ji\in A} x_{ji}^{w} - \sum_{j:ij\in A} x_{ij}^{w} = b_i^{w} = \begin{cases} 1, if\ i\in d(w) \\ -1, if\ i\in o(w) \\ 0, otherwise. \end{cases} ,\forall i, \forall w$$

$$x_{ij}^{w} + x_{ji}^{w} \le y_{ij}, \forall ij \in A, i < j, \forall w$$

RTND: Uncapacited Network Design (short)

$$\underset{\substack{x \in \{0,1\} \\ f \in R^+}}{Min.}\ c^f f + c^x x$$

$$Af^w = 1_i^w, \forall i, \forall w$$

$$f_{ij}^w + f_{ji}^w \le x_{ij}, \forall ij \in A, i < j, \forall w$$

$$c^x x \le c_{\max}$$

RTND: Uncapacited Network Design + node and edge location

$$\underset{\substack{x,y\in\{0,1\}\\ f\in R^{+}}}{Min}\ . c^{f} f + c^{x} x + c^{y} y$$

$$Af^{w} = 1_{i}^{w}, \forall i, \forall w$$

$$f_{ij}^{w} + f_{ji}^{w} \leq x_{ij}, \forall ij \in A, i < j, \forall w$$

$$f_{ij}^{w} \leq y_{i}, f_{ij}^{w} \leq y_{j}, \forall ij \in A, i < j, \forall w$$

$$c^{x} x + c^{y} y \leq c_{\max}$$

RTND: Uncapacited Network Design
+ node and edge location+line constraints

$$\underset{\substack{x,y\in\{0,1\}\\ f\in R^{+}}}{Min.}\ c^{f} f + c^{x} x + c^{y} y$$

$$Af^{w} = 1_{i}^{w}, \forall i, \forall w$$

$$f_{ij}^{w} + f_{ji}^{w} \leq \sum_{l\in L} x_{ij}^{l}, \forall ij \in A, i < j, \forall w$$

$$f_{o(w)j}^{w} \leq \sum_{l\in L} y_{j}^{l}, \forall o(w)j \in A, \forall w$$

$$f_{id(w)}^{w} \leq \sum_{l\in L} y_{i}^{l}, \forall i,d(w) \in A, \forall w$$

$$c^{x} x + c^{y} y \leq c_{\max}$$

$$Line\ \ constrs.(x, y, h), \forall l$$

RTND: Rapid Transit Network Design
Line Constraints

$x_{ij}^l \le y_i^l, \forall (i,j) \in A, i < j, \forall l \in L$	Links location
$x_{ij}^l \le y_j^l, \forall (i,j) \in A, i < j, \forall l \in L$	Link location
$x_{ij}^l = x_{ji}^l, \forall (i,j) \in A, \forall l \in L$	Directed links
$\sum_{\substack{j \in N(i) \\ i<j}} x_{ij}^l + \sum_{\substack{j \in N(i) \\ j<i}} x_{ji}^l \le 2, \forall i \in N, \forall l \in L$	Each node has not more than 2 edges
$h_l + \sum_{\substack{(i,j) \in A \\ i<j}} x_{ij}^l = \sum_{i \in N} y_i^l, \forall l \in L$	Number of edges is 1 less the number of nodes of each line.
$\sum_{\substack{(i,j) \in A \\ i<j}} x_{ij}^l(t) \le M_2 h_l(t), \forall l \in L, \forall t \in T$	$h_l = 1,\ if \sum_{(i,j) \in A, i<j} x_{ij}^l \ne 0$ and zero, otherwise
$\sum_{\substack{(i,j) \in A \\ i<j}} x_{ij}^l(t) \ge h_l(t), \forall l \in L, \forall t \in T$	
$\sum_{i \in B} \sum_{j \in B} x_{ij}^l \le \lvert B \rvert - 1, \forall B \subset N, \lvert B \rvert \ge 2, \forall l \in L$	Cycles by lines are not permited

RTND : Uncapacited Network Design + node & edge location+line constrs.+mode splitting

$$\underset{\substack{x,y,h,p\in\{0,1\}\\ f\in R^+}}{Max}.\ z = gp - c^f f - c^x x - c^y y$$

$$Af^w = 1_i^w, \forall i, \forall w : RDC(i,w)$$

$$\left.\begin{aligned} f_{ij}^w + f_{ji}^w &\le \sum_{l\in L} x_{ij}^l, \forall ij \in A, i<j, \forall w \\ f_{o(w)j}^w &\le \sum_{l\in L} y_j^l, \forall o(w)j \in A, \forall w \\ f_{id(w)}^w &\le \sum_{l\in L} y_i^l, \forall i,d(w) \in A, \forall w \end{aligned}\right\} LAC(w)$$

$$c^x x + c^y y \le c_{max} : \quad CCC$$

$$Line\ constrs.(x,y,h), \forall l : \quad LC(l)$$

$$\frac{1}{\lambda}\sum_{ij} d_{ij} f_{ij}^w - \mu u_w^{pri} \le M(1-p_w), \forall w : MDSC(w)$$

Rapid Transit Network Design Model

$$\underset{x,y,p,h,f\in\{0,1\}}{Min.} \quad z$$

$$subject\ to: RDC(i,w), LC(l),$$

$$MDSC(w), LAC(ij,w), CCC.$$

Rapid Transit Network Design
Model Size

$$R1: |N| = 6, |L| = 5, |W| = 30, |A| = 18$$

$$R2: |N| = 9, |L| = 5, |W| = 42, |A| = 36$$

$$R3: |N| = 20, |L| = 5, |W| = 380, |A| = 380$$

Binary Variable	x_{ij}^{l}	y_{i}^{l}	f_{ij}^{w}	p^{w}	h^{l}	Total
R1	75	30	450	30	5	590
R2	180	45	1512	42	5	1605
R3	950	100	72200	380	5	73635

Constraints	**RDC(i,w)**	**MSDC(w)**	**LC(l)**	**LAC(ij,w)**	**CCC**	**Total**
R1	180	6	270	630	1	1087
R2	378	42	600	2268	1	3289
R3	7600	380	2965	142400	1	153004

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Global Link Restoration
Survivable capacitated Network Design

Rajan and Atantürk 2003

$$\underset{\substack{y\in Z^+\\ x\in R^+}}{Min.} \sum_{w\in W}\sum_{\substack{ij\in A\\ i<j}} c_{ij}^{w}(x_{ij}^{w,0}+x_{ji}^{w,0})+\sum_{ij\in A} f_{ij}\, y_{ij}$$

$$\sum_{j:ji\in A} x_{ji}^{w,s}-\sum_{j:ij\in A} x_{ij}^{w,s}=b_i^{w}=\begin{cases} g_w, if\ i\in d(w)\\ -g_w, if\ i\in o(w)\\ 0, otherwise.\end{cases},\forall i\in N,\forall w\in W,\forall s\in S$$

$$\sum_{w\in W}(x_{ij}^{w,s}+x_{ji}^{w,s})\le q_{ij}\, y_{ij},\forall ij\in A\backslash\{s\}, i<j,\forall s\in S$$

Survivable Uncapacited Network Design size compared with Uncapacitaed Network Design size

	UNDP Constraints $\lvert N\rvert\lvert W\rvert+\lvert A\rvert/2$	UNDP Variables $\lvert A\rvert\lvert W\rvert+\lvert A\rvert/2$	SUNDP Constraints $\left(\lvert N\rvert\lvert W\rvert+\lvert A\rvert/2\right)\lvert S\rvert$	SUNDP Variables $\lvert A\rvert\lvert W\rvert\lvert S\rvert+\lvert A\rvert/2$
$\lvert N\rvert=20:\lvert W\rvert=380,$ $\lvert A\rvert=380,\lvert S\rvert=190$	7980	144590	1524180	27436190

Rapid Transit Network Design
ROBUSTNESS APPROACHES

- **Heuristic**
- **Reliability flow constraints**
  1. **Demand-arc flow**
  2. **Arc-flow**
  3. **Arc-demand**
- **Travelling time**
  1. **Arc failure maximum travelling time must be minimized.**
  2. **Maximum difference between arc failure travelling time and without failure traveling time must be minimized.**
- **Trip coverage**
  1. **Arc failure minimum trip coverage must be maximized.**

Rapid Transit Network Design with Demand-arc flow constraints

- Only a percentage of some demands are allowed to be routed trough the selected arcs.
- In arc failure event only a percentage of the demand is affected.

$$f_{ij}^{w} \le \frac{1}{r_{ij}^{w}}, \forall (i,j) \in E' \subset E, \forall w \in W' \subset W$$

$$f_{ij}^{w} \in [0,1], \forall (i,j) \in E' \subset E, \forall w \in W' \subset W$$

Uncapacitated Network Design with reliability flow constraints

$$\underset{\substack{y\in\{0,1\}\\ x\geq 0}}{Min.} \sum_{w\in W}\sum_{ij\in A} c_{ij}^{w} x_{ij}^{w} + \sum_{ij\in A} f_{ij}\, y_{ij}$$

$$\sum_{j:\,ji\in A} x_{ji}^{w} - \sum_{j:ij\in A} x_{ij}^{w} = b_{i}^{w} = \begin{cases} 1, if\ i\in d(w) \\ -1, if\ i\in o(w) \\ 0, otherwise. \end{cases}, \forall i, \forall w$$

$$x_{ij}^{w} + x_{ji}^{w} \leq y_{ij}, \forall ij\in A, i<j, \forall w\in W$$

$$x_{ij}^{w} \leq \frac{1}{r_{ij}^{w}}, \forall ij\in A, \forall w\in W$$

Capacitated Network Design with reliability flow constraints

$$\underset{\substack{y\in Z^+\\ x\in R^+}}{Min.} \sum_{w\in W}\sum_{ij\in A} c_{ij}^{w} x_{ij}^{w} + \sum_{ij\in A} f_{ij}\, y_{ij}$$

$$\sum_{j:\,ji\in A} x_{ji}^{w} - \sum_{j:ij\in A} x_{ij}^{w} = b_i^{w} = \begin{cases} g_w, if\ i\in d(w) \\ -g_w, if\ i\in o(w) \\ 0, otherwise. \end{cases}, \forall i, \forall w$$

$$\sum_{w\in W} (x_{ij}^{w} + x_{ji}^{w}) \le q_{ij}\, y_{ij}, \forall ij \in A, i < j$$

$$x_{ij}^{w} \le \frac{g_w}{r_{ij}^{w}}, \forall ij \in A, \forall w \in W$$

Rapid Transit Network Design: Arc failure difference maximum traveling time must be minimized

$$\underset{n \in N}{Min}\ \underset{ij \in A'}{Max}\left[T^{ij}(n) - T(n)\right]$$

$$T^{ij} = \sum_{w \in W} T_{ij}^{w} g_{w}$$

$$T_{ij}^{w} = p_{w}\left[\sum_{kl \neq ij} d_{kl} f_{kl}^{w} + (u_{ij}^{pri} + 0.2) f_{ij}^{w} + (u_{ji}^{pri} + 0.2) f_{ji}^{w}\right] + \mu u_{ij}^{pri}(1 - p_{w})$$

$$T = \sum_{w \in W} g_{w}\left(p_{w} u_{w}^{pub} + (1 - p_{w})\right) u_{w}^{pri}$$

T(n) is the total traveling time of network “n”.

T^{ij} (n) is the total traveling time if arc (I,j) fails at network n.

A lower bound on trip coverage of network is imposed, not lower than σ z*, z* being the trip coverage of the optimal network, σ in [0,1].

Global Link Restoration:
Reserve network dimensioning

Soriano et al. 2000

Survivable networks: All the demands can be met under the failure of any one of its links. Global link restoration: Link capacities over<whole the network that will allow rerouting under every link failure scenario.

Global link restoration in a survivable network carries several disadvantages:

- Formulation size
- Rerouting of disrupted as well as undisrupted flow in case of failure is harder to implement than rerouting only disrupted flow.
- Require sophisticated hardware and software and longer reconfiguration times.

Hybrid networks which are capacity-efficient and easy to restore under failure.

Global link restoration is very large model, **hierarchical restoration** schemes is popular.

1. Link capacity is determined for no-failure scenario. Capacity-efficient solution
2. Given the working (q_e) capacities, sufficient spare capacity is assigned to links, so disrupted flow can be safely rerouted in case of failure.

Survivable Restoration Telecommunication Networks

For each failure the capacity and the route for each demand must be determined.

Restoring strategies: *local or global. Grover and Doucette 2001.*

Local (link): rerouting only between broken link extremities. It is simpler to apply but capacity inefficient.

For a link failure the traffic is rerouted between the link extremities only.

Global (end-to-end/path): rerouting from all O/D of each disrupted demand. Apply to backbone networks.

For a link failure the traffic is rerouted for all the paths from O/D of each disrupted demand. It is more capacity efficient.

Global Link Restoration: Spare capacity network design

Soriano et al. 2000. Kennington and Whitler 1999. Gavish et al. 1989.

- S failed edge set. E edge set. S is a subset of E.
- R_w^s paths that link source o(w) and sink d(w) of demand w (without use the failed edge s).
- f_e cost of capacity of edge "e".
- W_s demand affected by fail "s".
- x_w^s demand "w" affected by fail edge "s". It's 1 in the incapacitated case-
- h_r flow at route "r".
- y_e spare capacity of edge "e".

$$\underset{\substack{y\in\{0,1\}\\ h\in Z^+}}{Min}.\sum_{e\in E} f_e y_e$$

$$\sum_{r\in R_w^s} h_r = x_w^s, \forall w \in W_s, \forall s \in S \subseteq E$$

$$\sum_{w\in W}\sum_{r\in R_w^s} h_r \delta_e^r \le q_e y_e, \forall e \in E, \forall s \in S \subseteq E, e \ne s$$

Working and spare capacity non distinguished: path flow restoration (path formulation)

- S failed edge set. W demand set. A arc set. N node set. S subset of A.
- W_s demand affected by fail in edge s. g_w demand commodity w.
- Y_{ij} capacity edge ij. C_{ij} unit capacity cost.
- R_W^0 working path set of w. R_w^s path set of w using edge s.
- $R^{s,w}$ path set of w using fail edge s. h_r path flow r.
- $R_{ij}^{s,w}$ path set of w belonging W_s using edge ij.

$$\underset{y_{ij}\geq 0, h_r \geq 0}{Min} . \sum_{ij\in A} c_{ij}\left(\sum_{w\in W}\sum_{r\in R_{ij}^{o,w}} h_r + y_{ij}\right)$$

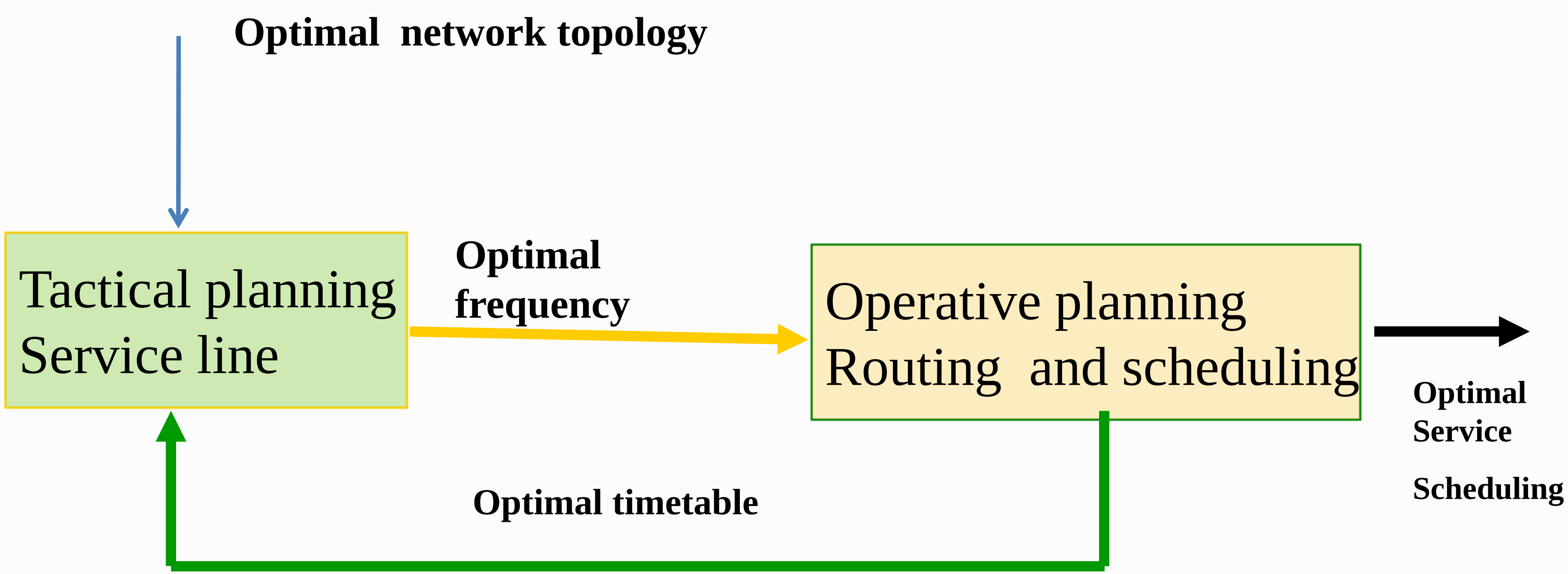
$$g_w = \sum_{r\in R^{0,w}} h_r, \forall w \in W ; x^{s,w} = \sum_{r\in R_s^{0,w}} h_r, \forall w \in W_s, \forall s \in S \subseteq E$$

$$x_{ij} = \sum_{w\in W}\sum_{r\in R_{ij}^{s,w}} h_r \leq y_{ij}, \forall ij \in E, \forall s \in S \subseteq E$$

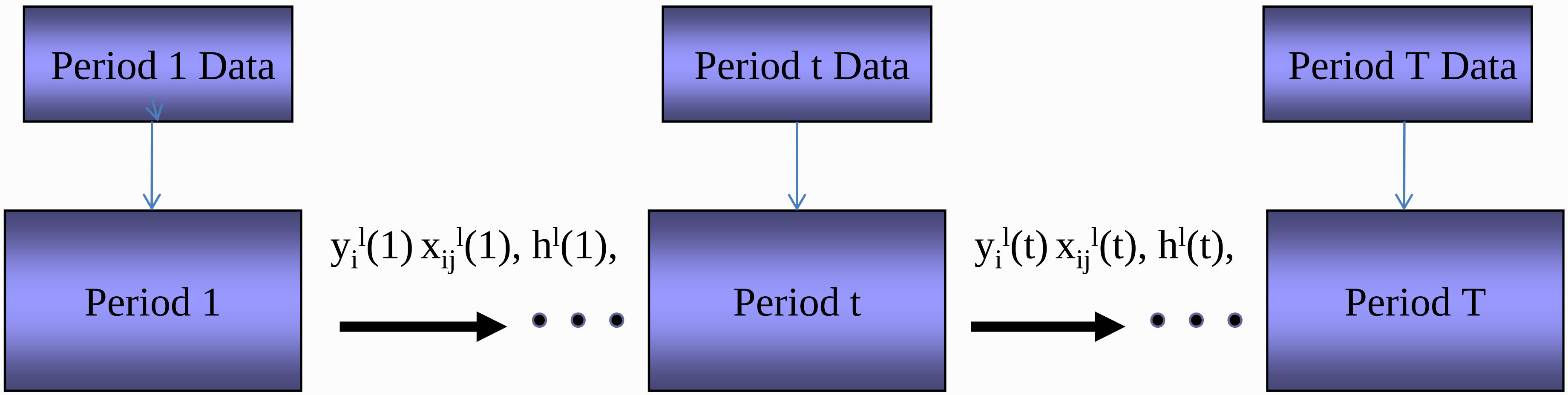
$$x^{s,w} = \sum_{r\in R^{s,w}} h_r, \forall w \in W_s, \forall s \in S \subseteq E$$

Lisser and Mahey 2006

Robustness Transit Network Design: Tactical and operative planning



Robustness Transit Network Design: Capacity expansion



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Airport Management

- Landing or Arrival Management (AMAN)
- Take-off or Departure Management (DMAN)
- Parking or Gate Management (GATEMAN)
- Taxi Planning (TP)
- Passenger and baggage management

Taxi Planning basic functions:

1. Landing: For a given landing instant time (exit from landing runway), determine optimal route and scheduling to parkings.

2. Downstream Take-off : If permission to leave parking is given at an instant time, determine optimal routes and scheduling to reach a take-off runway.

Taxi Planning network:

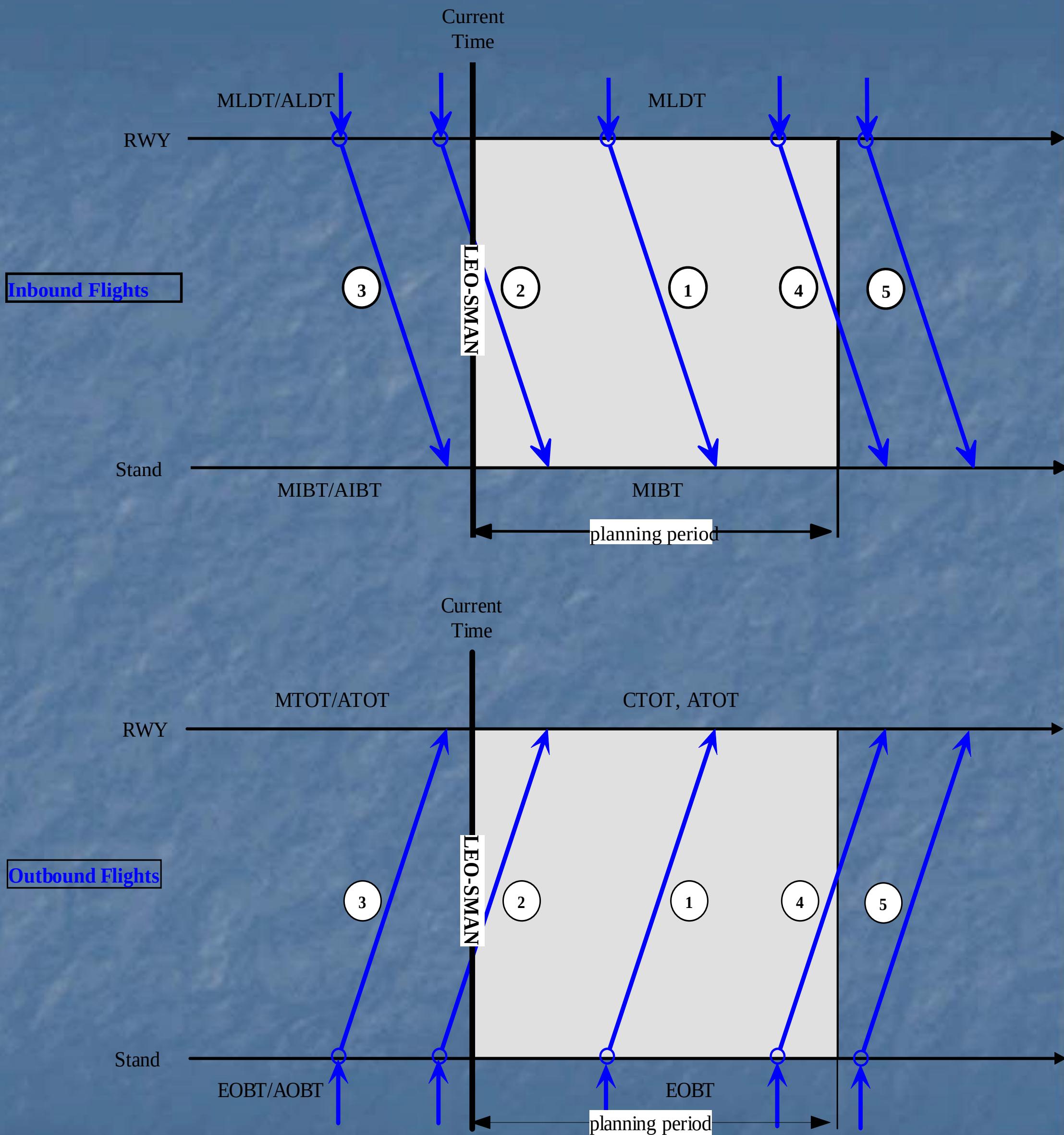
A directed network G=(N,A)

- A node $i \in N$ can be a parking, a holding area, a intersection of two or more taxiways, or a runway header or exit gate, etc.
- An arc $(i,j) \in A$ connecting nodes *i*, *j*, typically represents a physical taxiway, an entrance- and exit-ways to-from a stand, etc.

For time issues within the planning period, we replicate the network over time by an indexed set T

We consider a set of flights W, where $w \in W$ represents a specific flight

- Fixed Planning Period
  - ex: 30 mins
- Routes-Scheduling
  - 1: To optimise
  - 2: To optimise/Fixed
  - 3,5: not considered
  - 4: To optimise



Landing

Input:

- Origin Node, $o(w) \in N^{ER}$
- Time instant at Origin, $t(w) \in T$
- Destination parking, $d(w) \in N^{P}$

Output:

- Optimal Routing and sequencing.
- Optimal arriving at parking for aircraft.

  w, $OAPH^{W} \in T$

Downstream Take-off

Input:

- Origin parking, $o(w) \in$ N^{P}
- Time instant to exit from parking, $t(w) \in T$
- Destination runway, $d(w) \in$ N^{AR}

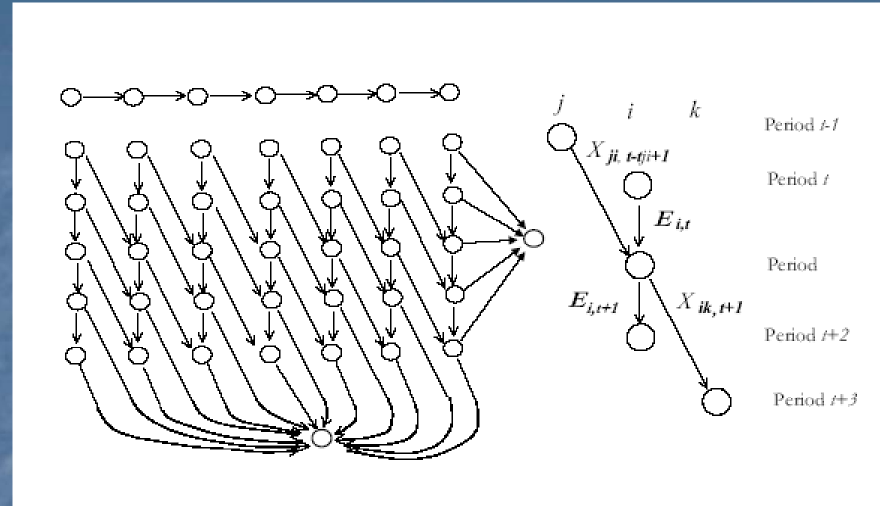
Output:

- Optimal routing and sequencing.
- Optimal instant time for taking-off, ODTH^{w} $\in T$

VARIABLES

$E_{i,t}^{w}$: Wait for aircraft w, at node i and period t.

$X_{ij,t}^{w}$: Move ahead for aircraft w, on link (i,j) and period t.



Balance constraints at nodes

$$B^{w}U^{w} = b^{w}, \forall w \in W$$

$$\sum_{j \in T^{*}(i)} X_{j,i,t-t_{ji}+1}^{w} + E_{i,t}^{w} - \sum_{j \in F^{*}(i)} X_{i,j,t+1}^{w} - E_{i,t+1}^{w} = b_{i,t+1}^{w},$$

$$\forall i \in N^{*}, \forall t \in T, \forall w \in W$$

TP: Objective Function

Weighted on the total ground's travel time

$$TP: \tau(U) = \underset{U \in \{0,1\}}{Min.} \sum_{w \in W} A^w U^w$$

$$\tau(X,E) = \sum_{w \in W} \sum_{t \geq t(w)} \lambda^w \left(\sum_{ij \in A} t_{ij} X^w_{ij,t} + \sum_{i \in N^W} E^w_{i,t} \right) + \sum_{w \in W} \sum_{i \in N^W} r_i^w E^w_{i,|T|}$$

Inside P.P. + Outside P.P.

Limited capacity at nodes:

$$M^w U^w \le q_a, \forall a \in A_1^*$$

$$\sum_{w \in W} e_w E_{i,t}^w \le q_i, \forall i \in N, \forall t \in T$$

Other capacity constraints:

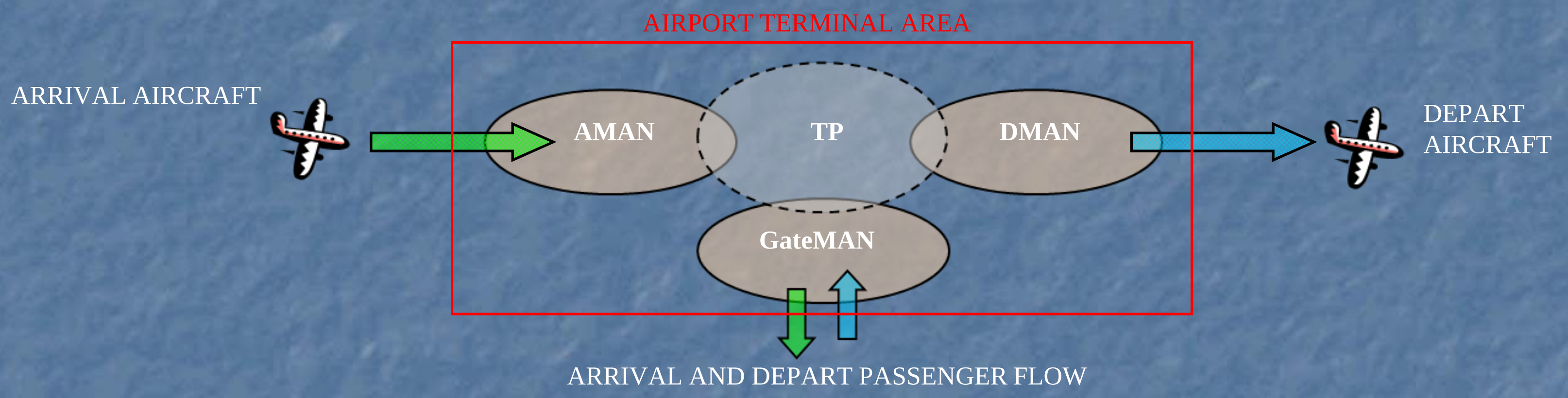
- Stopping is prohibited at some nodes.
- Some nodes can have only one aircraft waiting.
- Access to a node is limited to one aircraft at a time.
- Overtaking is avoided in taxiways
- Stands: Avoid the arrival of landing traffic to a stand and/or stand exit area if it is still occupied by a departing flight
- Runway blocking times depending on aircraft characteristics.

  (Landing and take-off runways, Mixed runways)
- Specified Time-windows for some taking-offs.

Indice

- ✓ **Rail Transportation Planning**
- ✓ **Uncapacitated and capacitated network design models**
- ✓ **Uncapacitated and capacitated service network design models**
- ✓ **Rapid Transit Network Design (RTND)**
- ✓ **Robustness RTND and capacitated network design models**
- ✓ **Taxi Planning: Routing and scheduling models**
- ✓ **Airport Terminal Area Design**

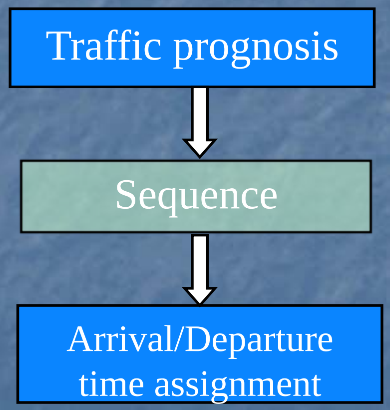
Integrating AMAN, TP, DMAN and GateMAN on Terminal Manoeuvring Area

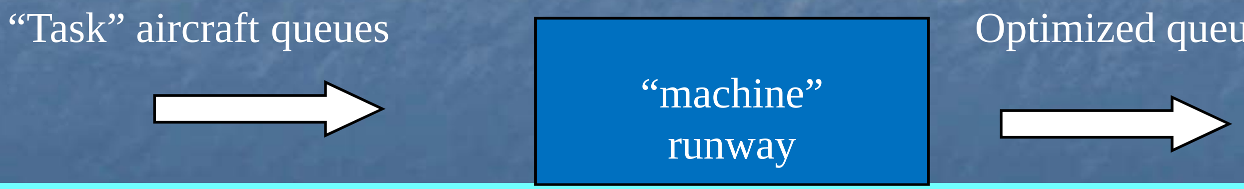


TERMINAL AREA SCHEDULING

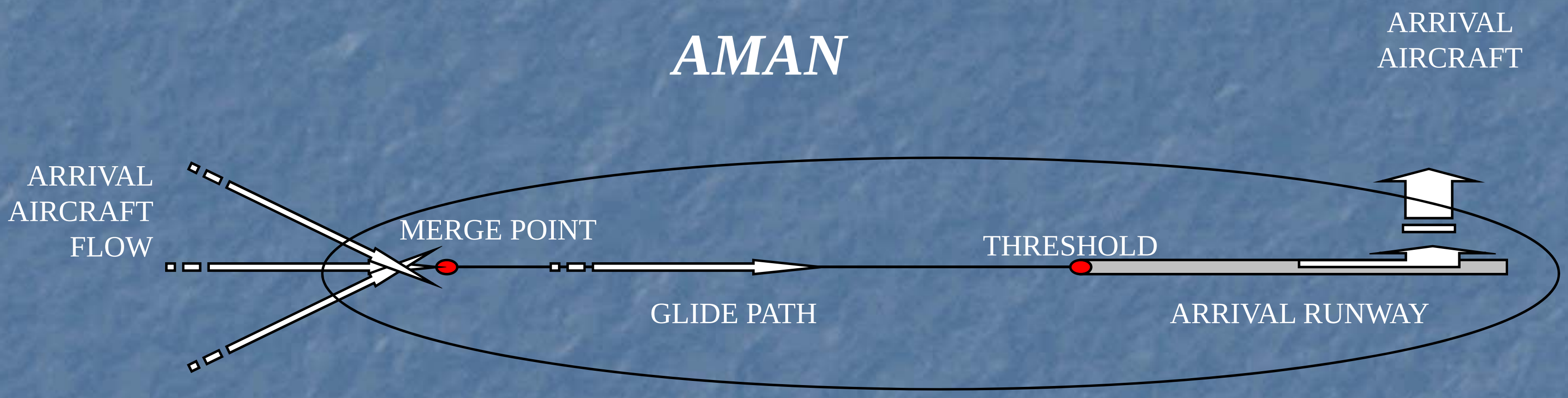
Introduction:

- "Scheduling on Airport Terminal Manoeuvring Area" is modelized by techniques of "*job scheduling*".
- Discrete optimization based on tasks management looking for the optimal task sequences.
- AMAN – DMAN:
  - Static model
  - The runways are considered as machines
  - The aircraft are considered tasks to process

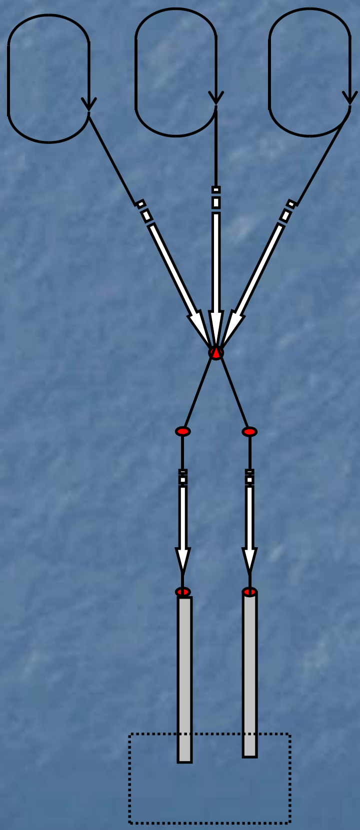




AMAN DIAGRAM



Arrival Management (AMAN)



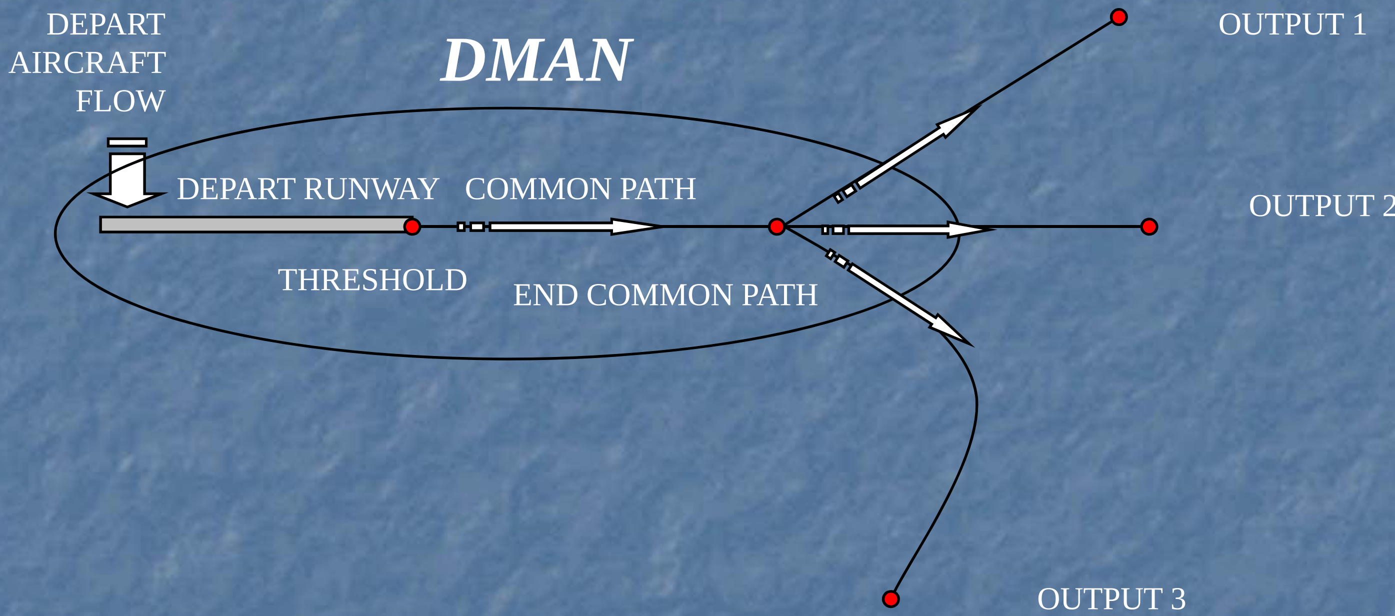
FIXES

MERGE POINT

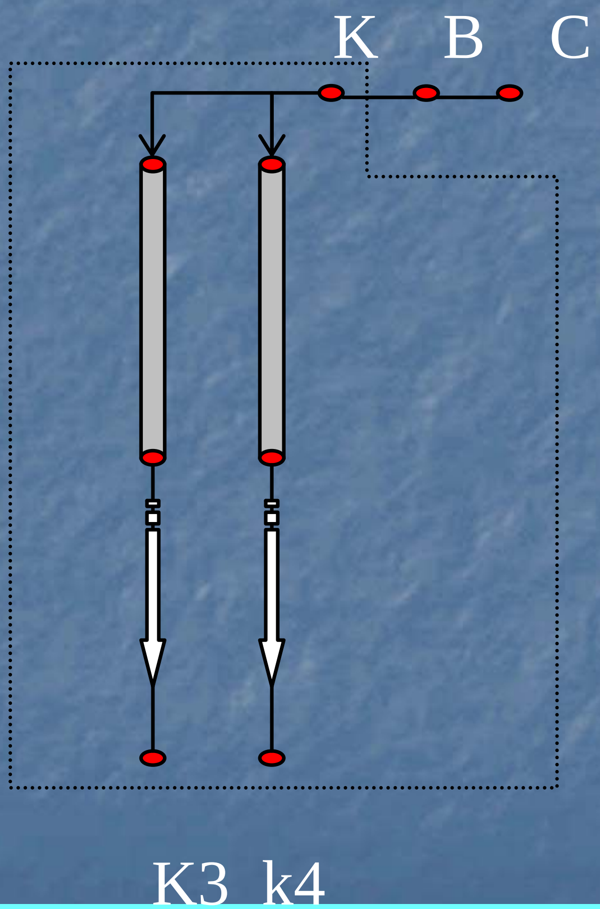
ARRIVAL RUNWAY IN PARALLEL

K1 k2

DMAN DIAGRAM

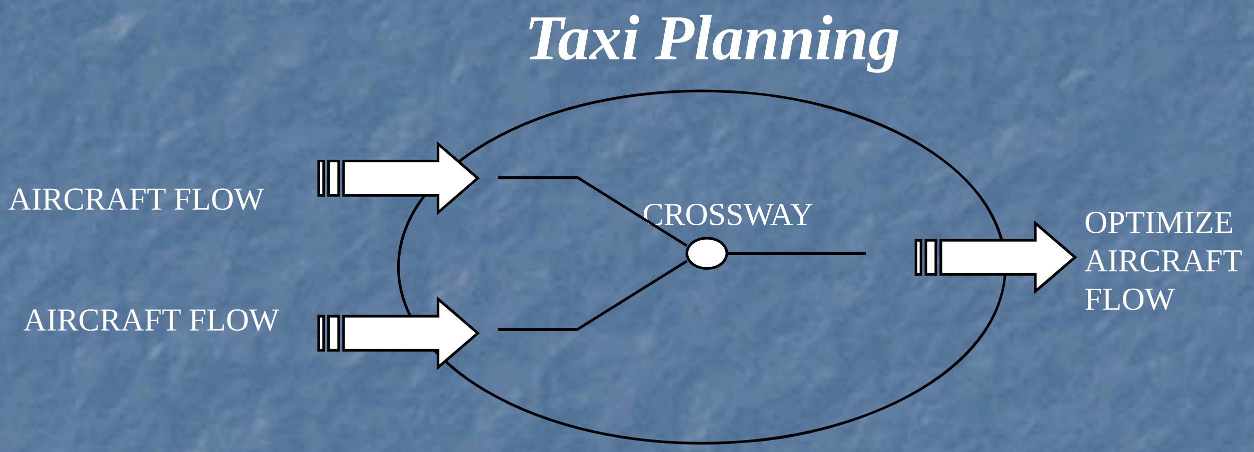


Depart Management (DMAN)



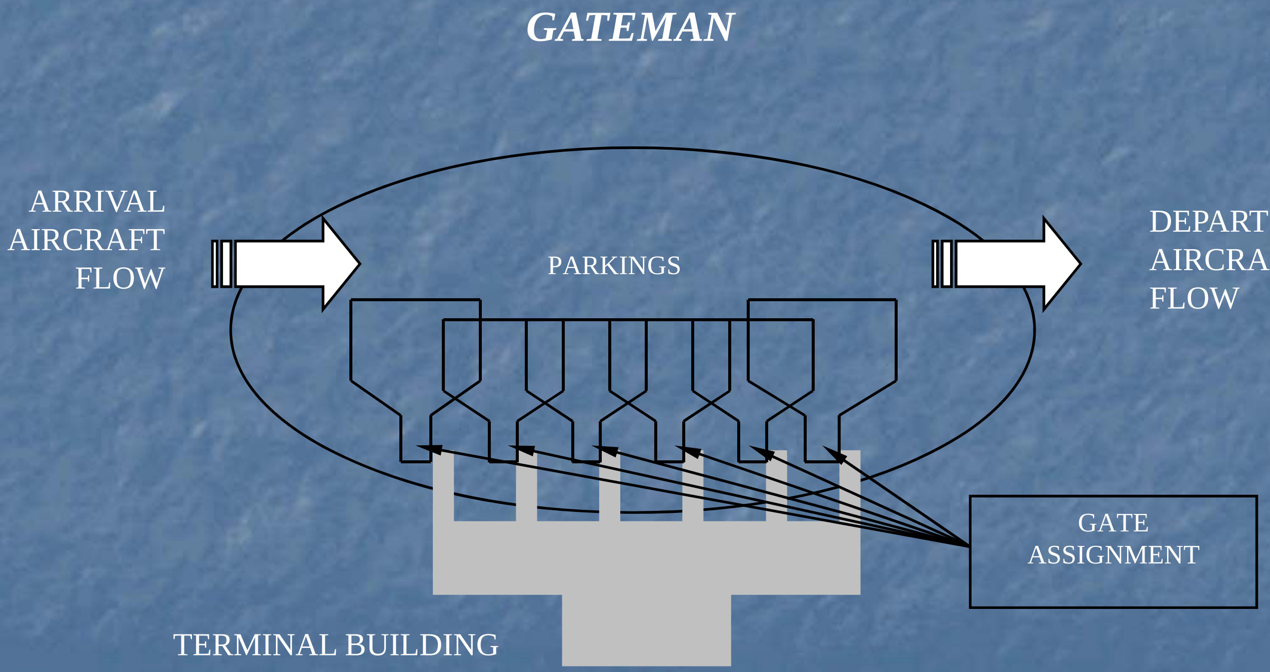
DEPART RUNWAY IN PARALLEL

TP DIAGRAM



Taxi Planning
AIRCRAFT FLOW
AIRCRAFT FLOW
CROSSWAY
OPTIMIZE
AIRCRAFT
FLOW

GATEMAN DIAGRAM



Alguna pregunta?


Ambrosio Morales Jiménez

Gracias!