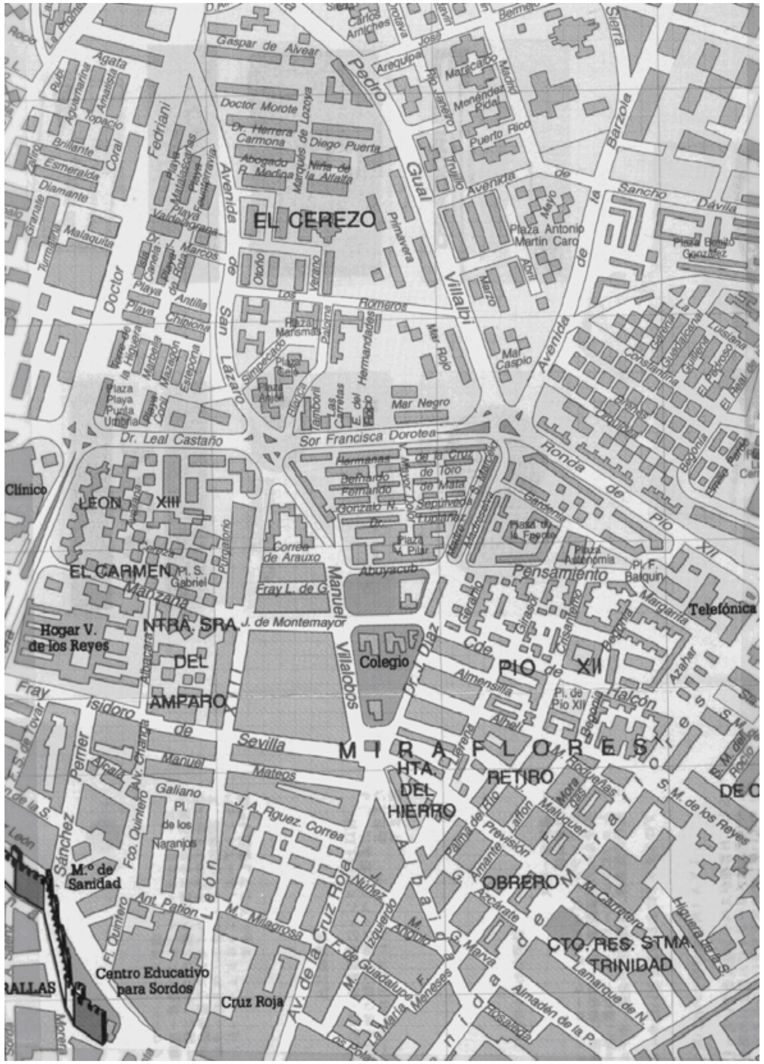
Ingeniería e infraestructura de los Transportes
Escuela Superior de Ingenieros, Universidad de Sevilla
Francisco García Benítez

MODELIZACION DEL TRÁFICO utilizando técnicas de DISCRETIZACIÓN DEL CONTINUO

Indice

- *Interés de este tipo de modelizaciones*
- *Bases y fundamentos: Flujo de tráfico*
- *Ejemplos*
- *Líneas investigadas*
- *Conclusiones*

Motivación

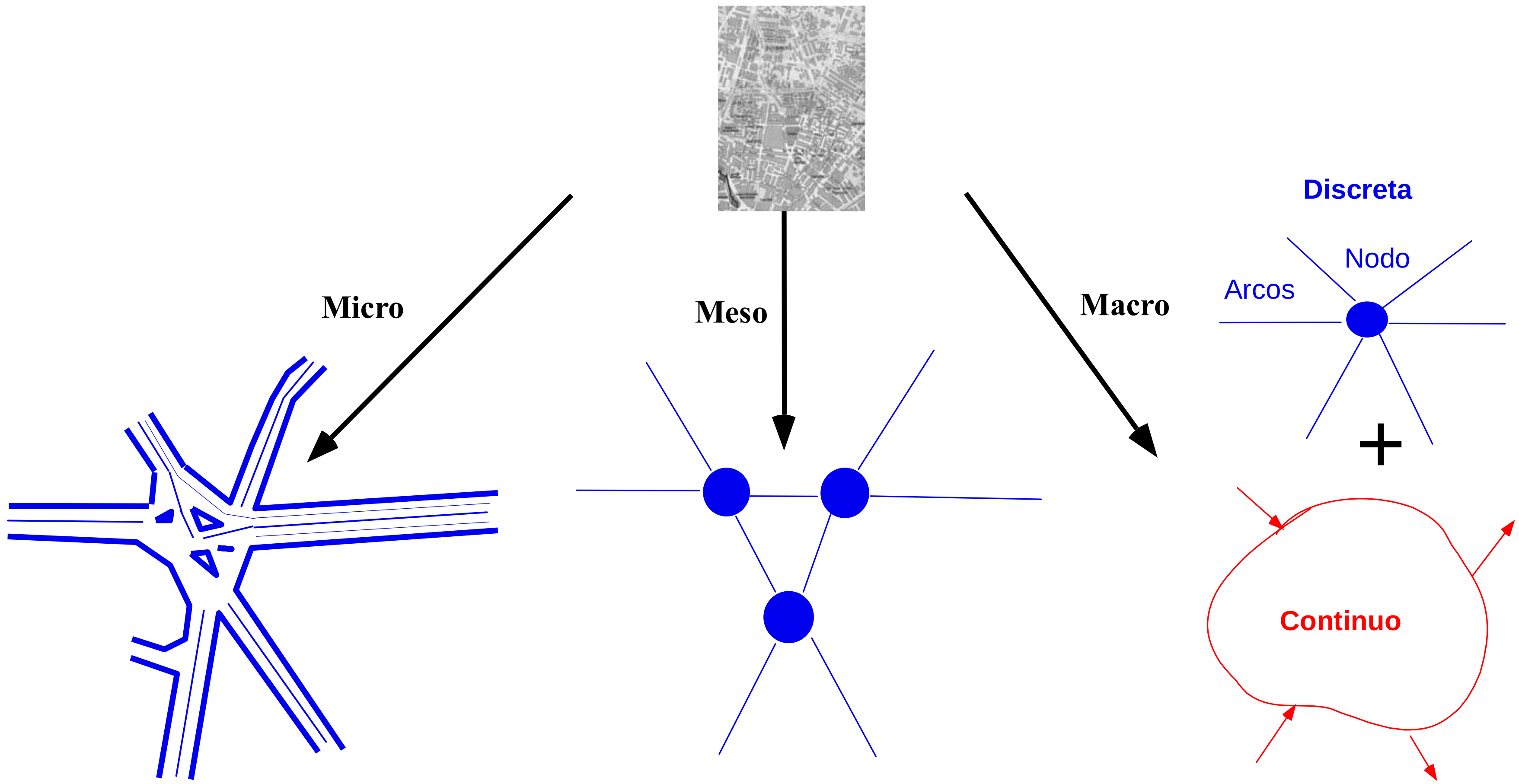


Areas urbanas densas

¿ Es posible simular areas urbanas densas reales?

¿Cuál podría ser la metodología más adecuada ?

Modelos disponibles actualmente

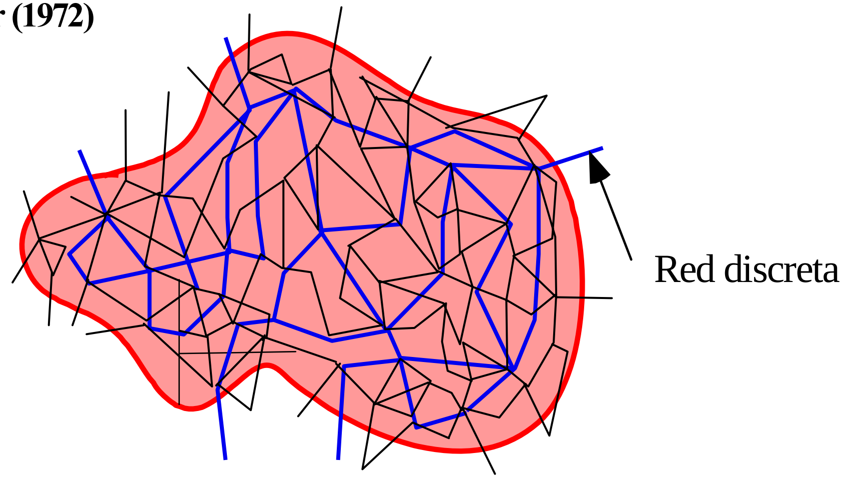


Micro	Meso	Macro
x_n , v_n , a_n	$\cancel{x_n , v_n , a_n}$	$\cancel{x_n , v_n , a_n}$
p_n	p_n	$\cancel{p_n}$
Q , K , V	Q , K , V	Q , K , V

Metodologías actuales

- **Técnicas utilizadas durante la década de los 80 y 90**
- **Aproximaciones basadas en "Redes Discretas"**

- R. B. Potts, R. M Oliver (1972)



Región urbana

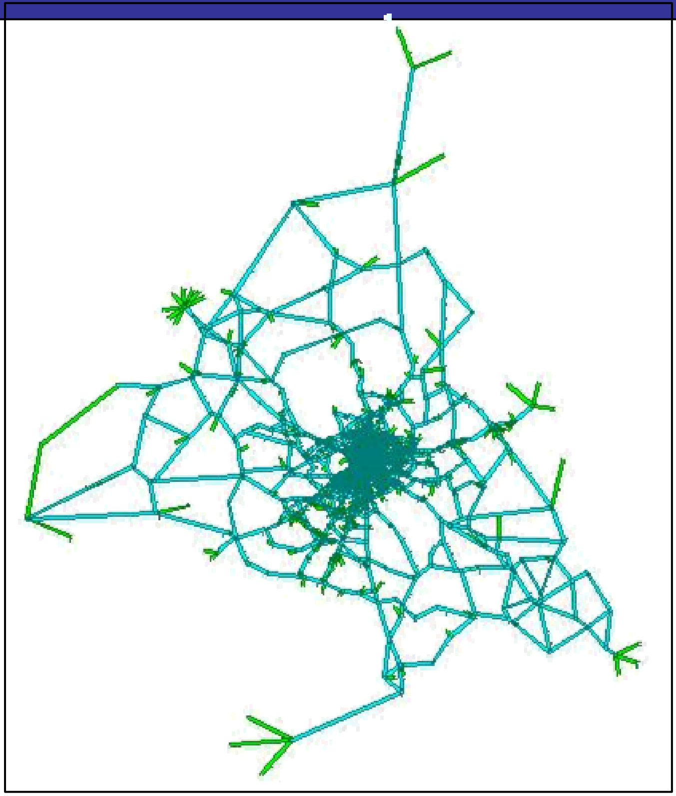
VENTAJAS

- Modelado realista (dependiendo del grado de representatividad de la red)
- Facilidad de modelado de acciones, intervenciones, modificaciones, obras

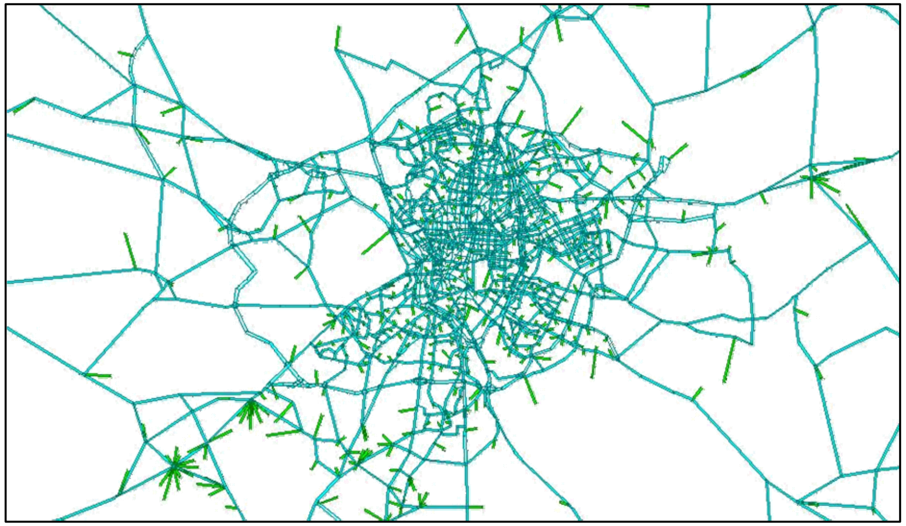
DESVENTAJAS

- Modelado costoso
- Mantenimiento y actualizaciones costosas
- Dificultad en la obtención de soluciones (interpretables, robustas)
- Exigencia de sistemas informáticos y expertos dedicados

Métodos actuales



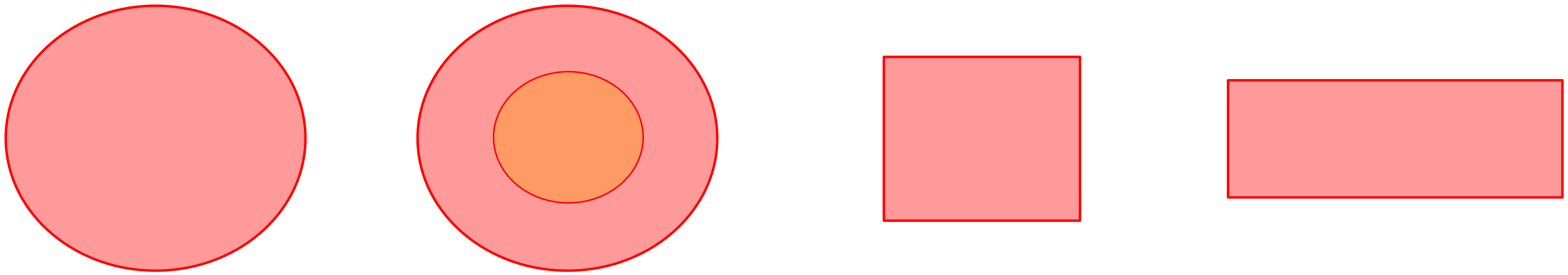
Arcos: 8,080
Nodos: 3,641
Centroides: 677



Interés: Métodos clásicos de continuo

- **Técnicas utilizadas durante las décadas de los 70 y 80**
- **Approximación basada en el "Modelado del Continuo"**

- G. Newell (1980) **- R. Vaughan (1987)**



VENTAJAS

• Modelado fácil
  - ➢ pocos datos
  - ➢ modelos sencillos
  - ➢ soluciones simples

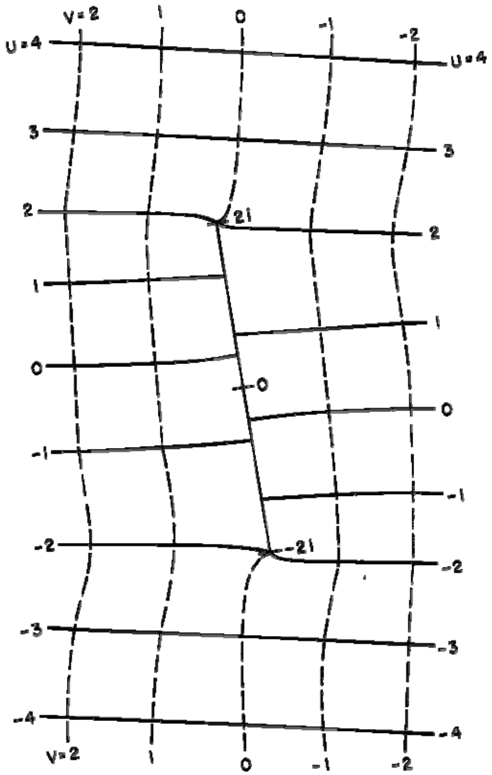
DESVENTAJAS

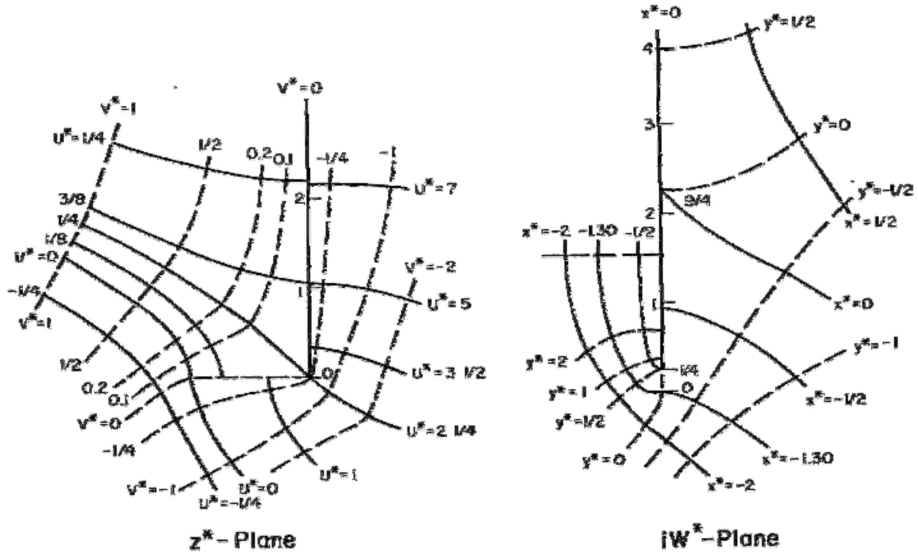
• Limitado realismo
• Capacidad reducida para el modelado de modificaciones e intervenciones
• Dificultad en validar los resultados con la realidad

Interés:

Métodos clásicos de continuo

From G. Newell (1993)



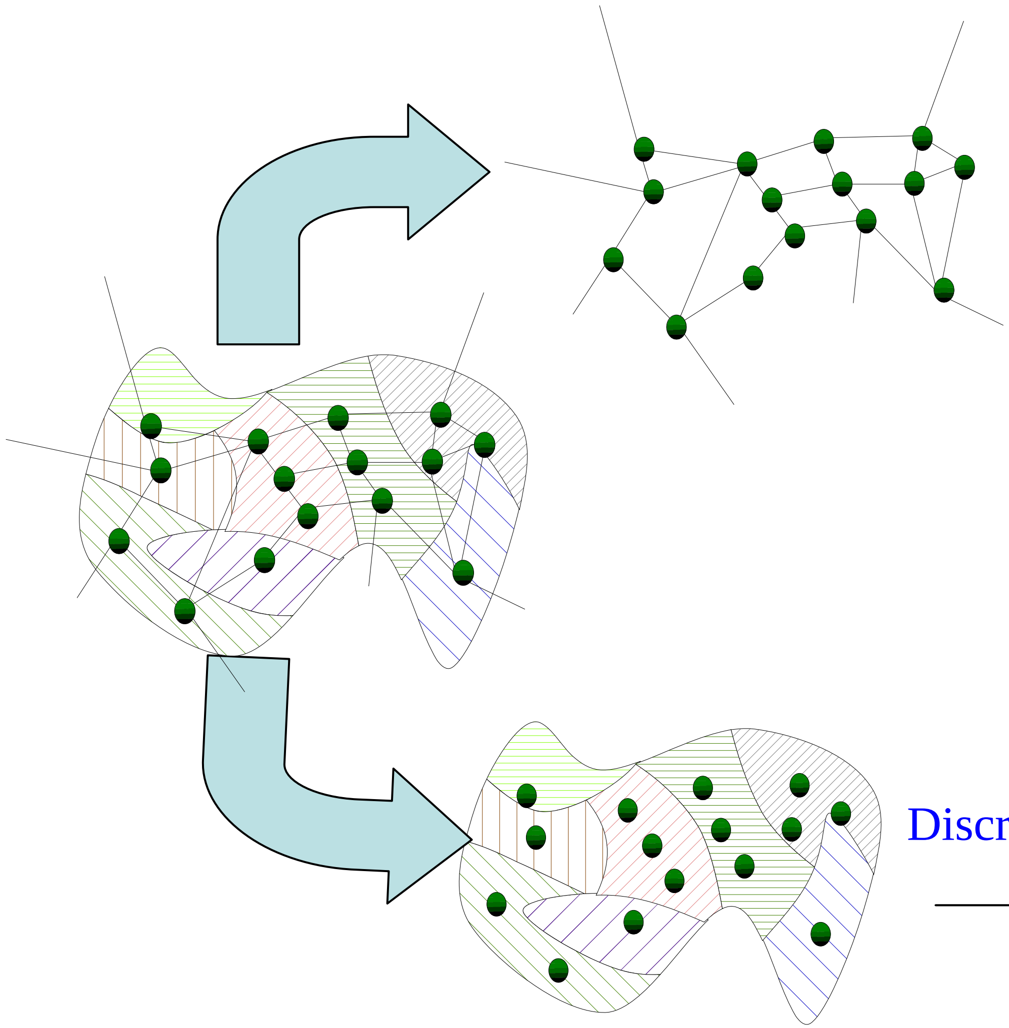


Flow around a linear obstruction

Flow around the end of a slit

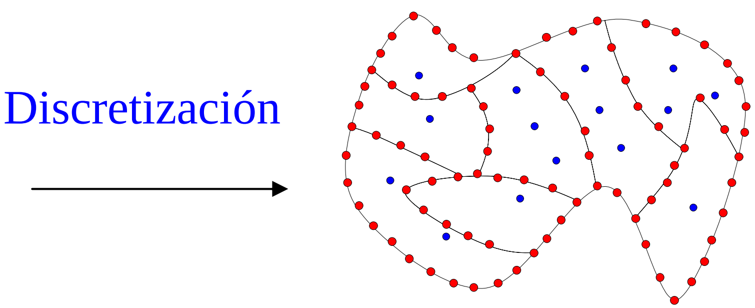
Modelos “académicos” experimentales

- Técnicas utilizadas desde 1992- ...
- Aproximaciones basadas en “Redes Densas”

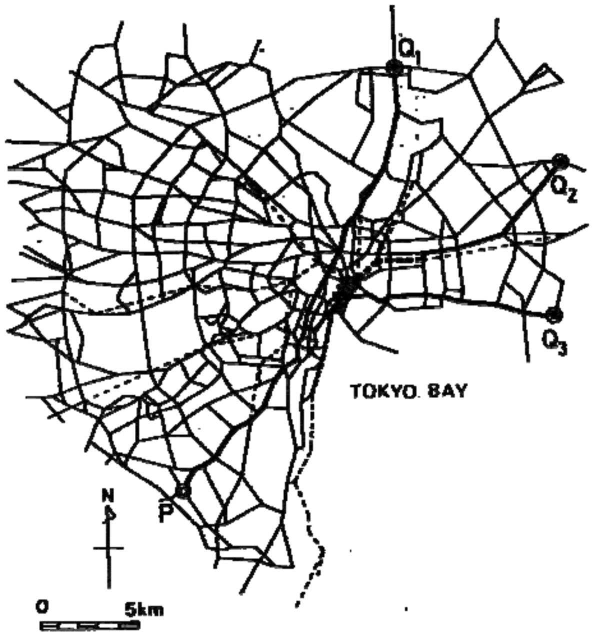


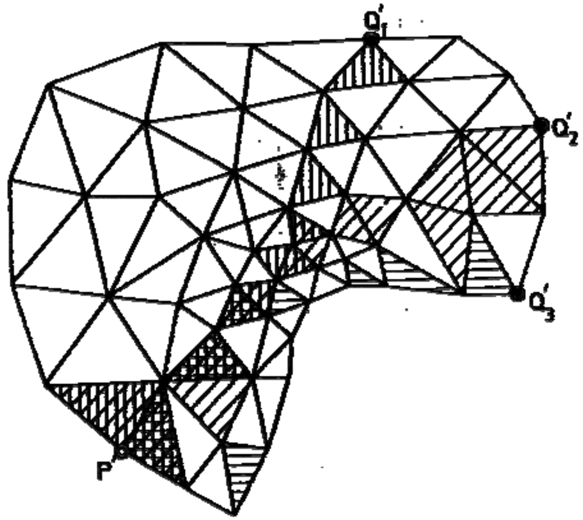
Teoría de redes

- A. Taguchi, M. Iri (1982)
- H. Yang, S. Yagar, Y. Iida (1994)
- S. C. Wong (1998 – actualidad)

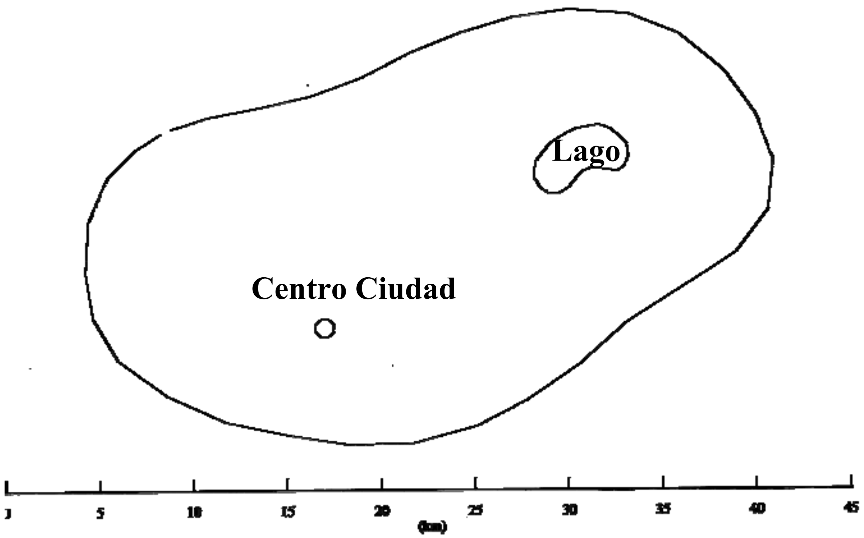


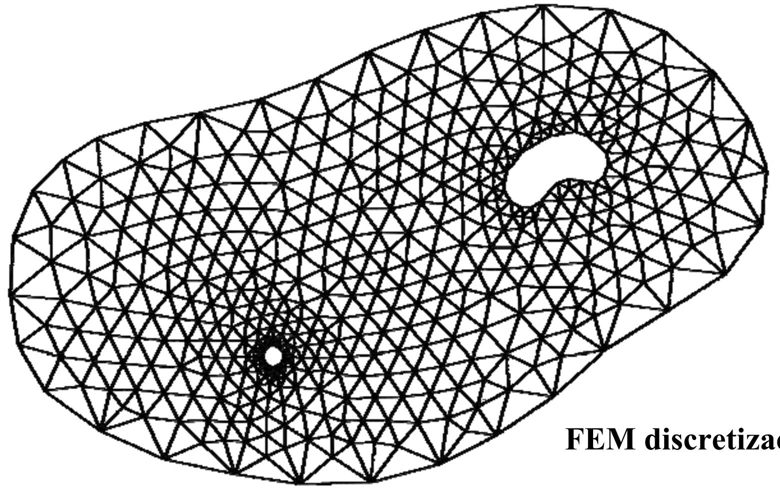
From A. Taguchi, M. Iri (1982)



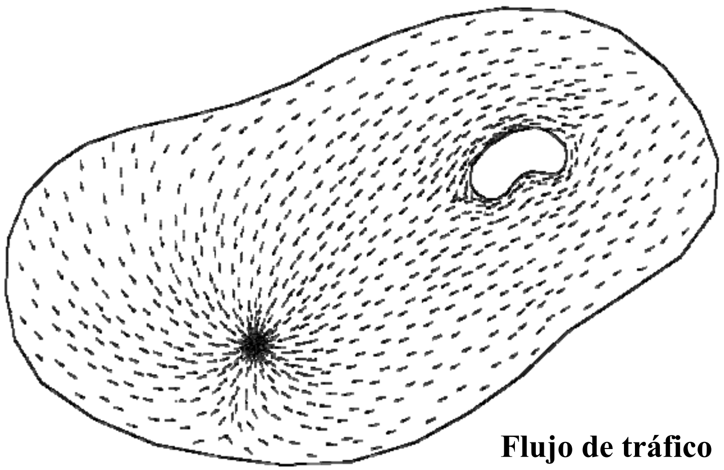


From S.C. Wong (1998)

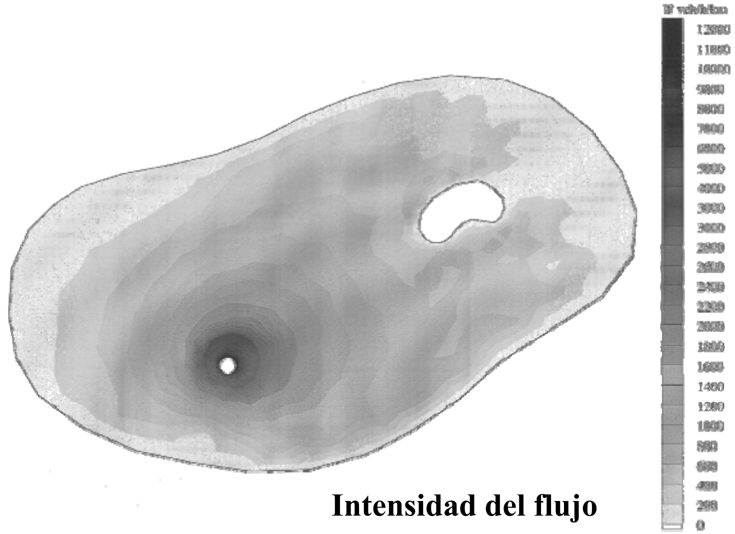




FEM discretización

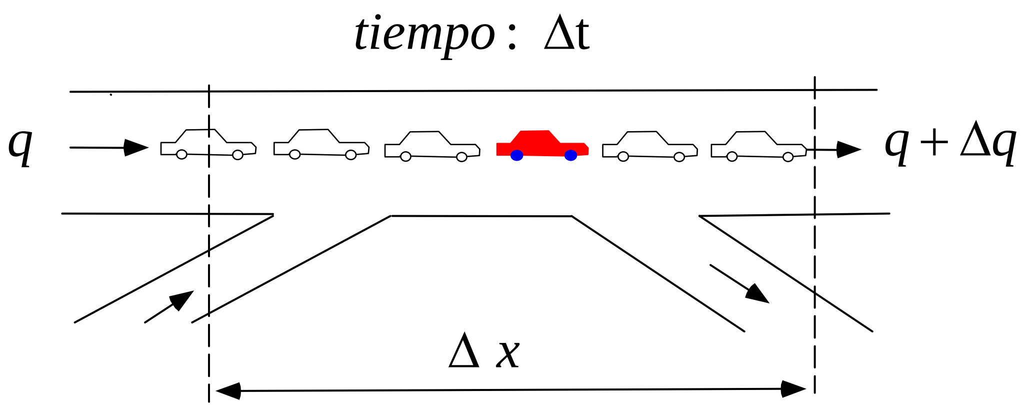


Flujo de tráfico



Intensidad del flujo

Flujo de tráfico uni-dimensional



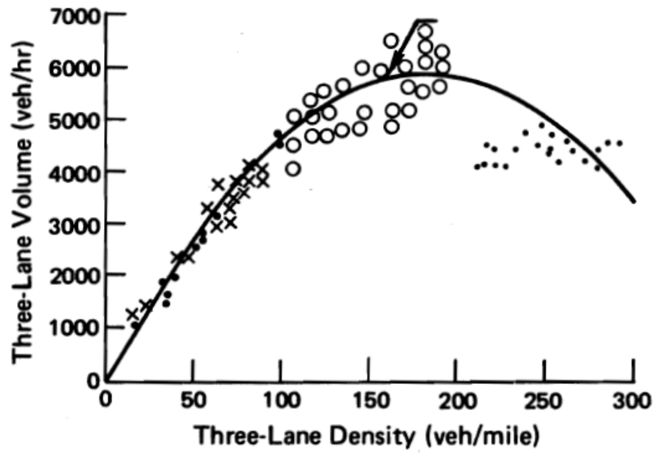
$$\frac{(q+\Delta q)-q}{\Delta x} + salida - entradas = -\frac{\Delta K}{\Delta t}$$

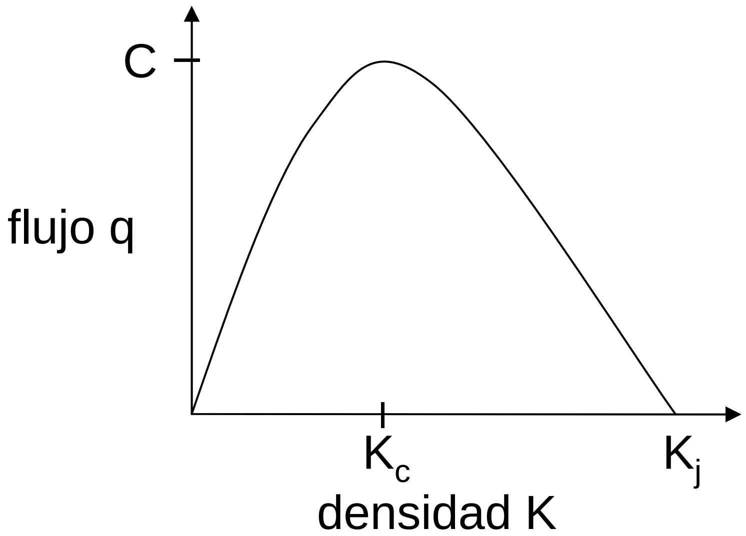
$$\Downarrow$$

$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

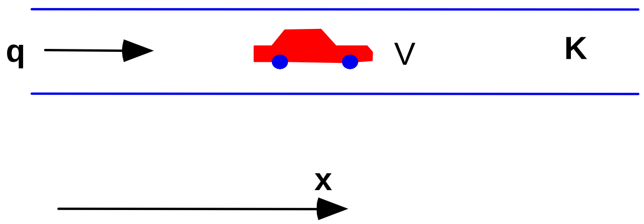
Lighthill-Whitham, 1955; Richards, 1956: Modelo 1er orden

Flujo de tráfico uni-dimensional





Flujo de tráfico uni-dimensional

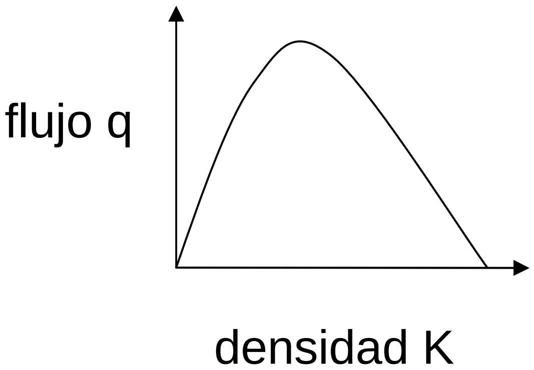


Ecuación de continuidad

$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

Lighthill-Whitham, 1955: Modelo 1er orden

Diagrama fundamental del tráfico

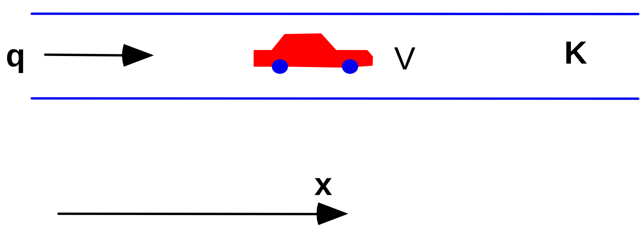


$$q(x,t) = Q(K(x,t))$$

$$V_e = \frac{q}{K} = V(K(x,t))$$

$$q(x,t) = K(x,t) \cdot V(x,t)$$

Flujo de tráfico uni-dimensional

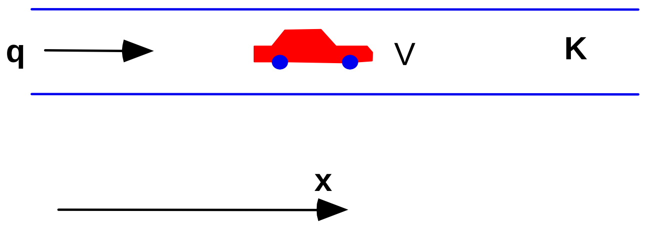


Modelo simple de 1er orden

$$\frac{\partial K(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=\theta(x,t)$$

$$q(x,t)=K(x,t)\cdot V(x,t)$$

Flujo de tráfico uni-dimensional



Modelo simple de 1er orden

$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

$$q(x,t) = K(x,t) \cdot V(x,t)$$

Modelo de 2º orden

$$\cancel{V_e = \frac{q}{K} = V(K(x,t))} \quad \Rightarrow \quad a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = \frac{V_e - V}{T} - \frac{\nu}{T}\frac{1}{K}\frac{\partial K}{\partial x}$$

Payne, 1971

Flujo de tráfico uni-dimensional

Modelo simple difusivo

$$q = Q(K(x,t)) - \mu \frac{\partial K}{\partial x}$$

$$v = V(K(x,t)) - \mu \frac{1}{K} \frac{\partial K}{\partial x}$$

Whitham, 1974

Modelo simple de 1er orden

$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

$$q(x,t) = K(x,t) \cdot V(x,t)$$

Modelo de 2º orden

$$a = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = \frac{V_e - V}{T} - \frac{\nu}{T} \frac{1}{K} \frac{\partial K}{\partial x}$$

Flujo de tráfico uni-dimensional

Modelo simple difusivo

$$q = Q(K(x,t)) - \mu \frac{\partial K}{\partial x}$$

$$v = V(K(x,t)) - \mu \frac{1}{K} \frac{\partial K}{\partial x}$$

Whitham, 1974

Modelo simple de 1^er orden

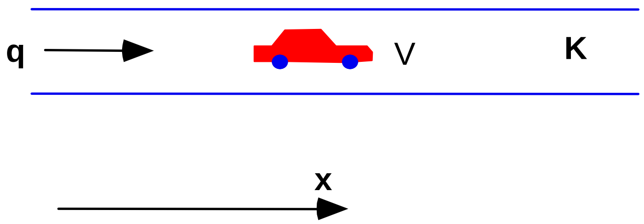
$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

$$q(x,t) = K(x,t) \cdot V(x,t)$$

Modelo de 2º orden

$$a = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = \frac{V_e - V}{T} - \frac{\nu}{T} \frac{1}{K} \frac{\partial K}{\partial x}$$

Flujo de tráfico uni-dimensional

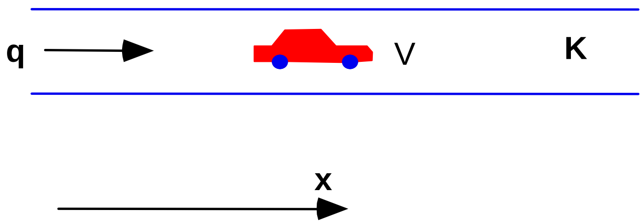


Modelo simple de 1er orden difusivo

$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

$$q(x,t) = K(x,t) \cdot V(x,t) - \mu \frac{\partial K}{\partial x}$$

Flujo de tráfico uni-dimensional

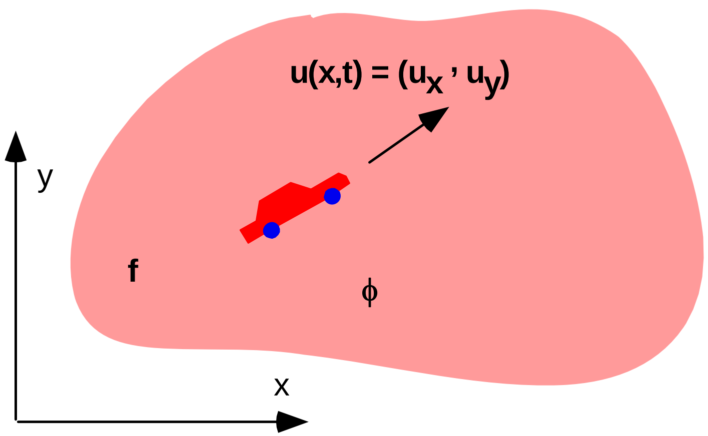


Modelo simple de 1er orden difusivo

$$\frac{\partial K(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \theta(x,t)$$

$$q(x,t) = K(x,t)\cdot V(x,t) - \mu \frac{\partial K}{\partial x}$$

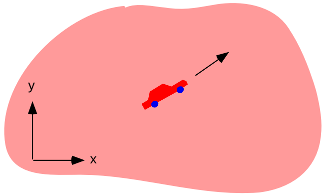
Flujo de tráfico bi-dimensional



$$\frac{\partial \phi(x,t)}{\partial t} + \nabla\cdot\mathbf{f}(\mathbf{x},t) = \rho(\mathbf{x},t)$$

$$\mathbf{f}(\mathbf{x},t) = \underbrace{\phi(x,t)\; u(x,t)}_{convectiva} \underbrace{-\mu\; \nabla\phi(x,t)}_{difusiva}$$

Flujo de tráfico bi-dimensional



Ecuación de advección-difusión

$$\frac{\partial \phi(x,t)}{\partial t} - \mu \nabla^2 \phi(x,t) + \nabla[u(x,t)\phi(x,t)] = \rho(x,t)$$

Ecuación de convección-difusión

$$\frac{\partial \phi(x,t)}{\partial t} - \mu \nabla^2 \phi(x,t) + u(x,t) \cdot \nabla \phi(x,t) + k\phi(x,t) = \rho(x,t)$$

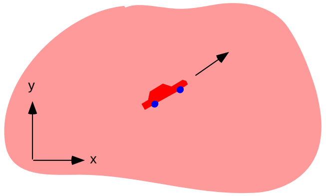
$$u(x,t) = \overline{u}(t) + \tilde{u}(x,t)$$

$$\nabla \cdot \overline{u}(t) = 0$$

$$\frac{\partial \phi(x,t)}{\partial t} - \mu \nabla^2 \phi(x,t) + \overline{u}(t) \nabla \phi(x,t) + \nabla[\tilde{u}(x,t)\phi(x,t)] = \rho(x,t)$$

Flujo de tráfico bi-dimensional

Ecuación de advección-difusión



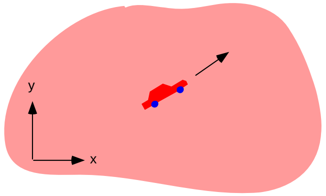
$$\frac{\partial \phi(x,t)}{\partial t} - \mu \nabla^2 \phi(x,t) + \bar{u}(t) \nabla \phi(x,t) + \nabla[\tilde{u}(x,t)\phi(x,t)] = \rho(x,t)$$

Ecuación de convección-difusión

$$\frac{\partial \phi(x,t)}{\partial t} - \mu \nabla^2 \phi(x,t) + u(x,t) \cdot \nabla \phi(x,t) + k\phi(x,t) = \rho(x,t)$$

Flujo de tráfico bi-dimensional

Ecuación de advección-difusión



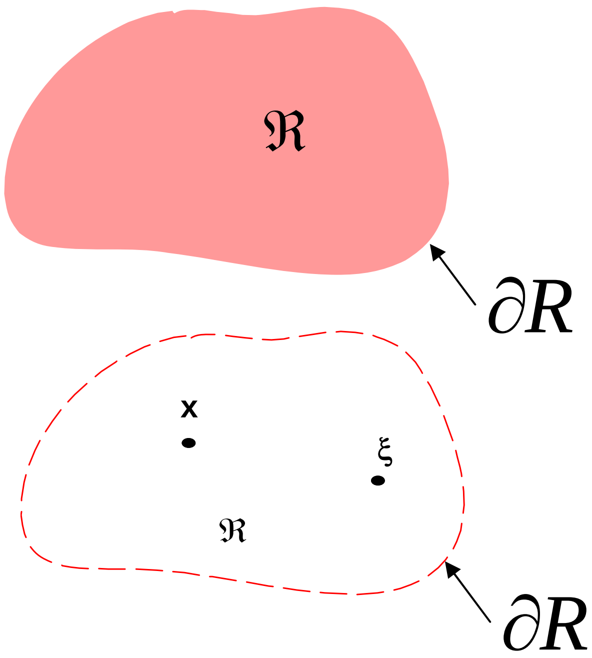
$$\mu\nabla^2\phi(x,t) - \bar{u}(t)\nabla\phi(x,t) = \frac{\partial\phi(x,t)}{\partial t} + \nabla[\tilde{u}(x,t)\phi(x,t)] - \rho(x,t)$$

Ecuación de convección-difusión

$$\mu\nabla^2\phi(x,t) - u(x,t)\cdot\nabla\phi(x,t) - k\phi(x,t) = \frac{\partial\phi(x,t)}{\partial t} - \rho(x,t)$$

$$L[\phi] = \chi(x,t)$$

Bases y fundamentos:
Método de los elementos de contorno (BIEM)

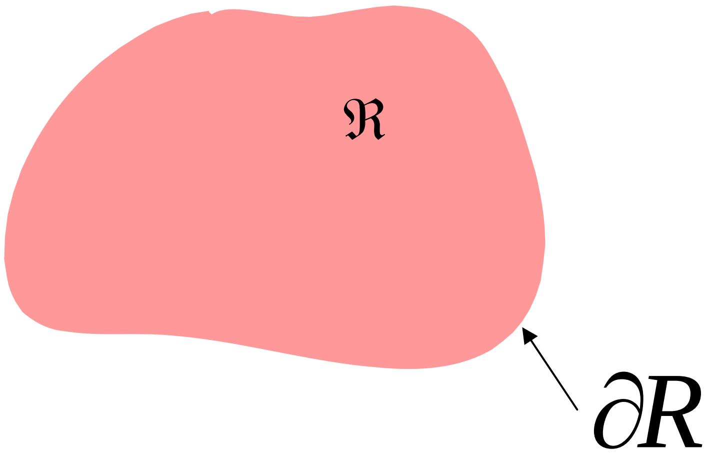


$$L\left[\phi\right] = \chi(x,t)$$

$$L^{*}\left[\psi\right] = -\delta(x,\xi)$$

$$c(\theta)\phi(\xi) + \int_{\partial R}\left[h(x,\xi)\ \phi(x) - g(x,\xi)\frac{\partial\phi(x)}{\partial n}\right] d\Gamma_x = \int_{R} \chi(x,t)\psi(x,\xi,t)\ dR_x$$

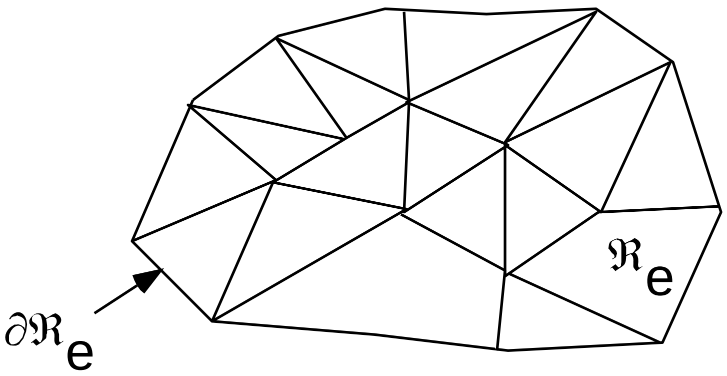
Bases y fundamentos:
Flujo de tráfico bi-dimensional y uni-bien



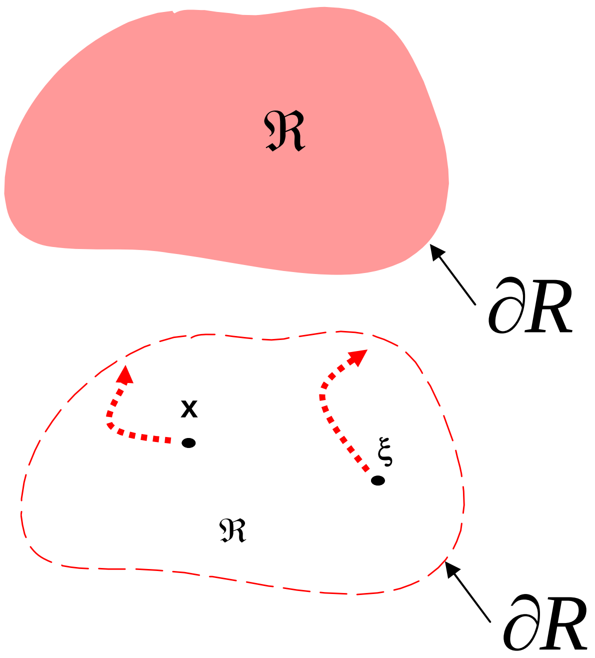
$$\int_{\partial R} (.)\, d\Gamma_x \qquad\qquad \int_{R} (.)\, dR_x$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\sum_e \int_{\partial R_e} (.)\, d\Gamma_e \qquad\qquad \sum_e \int_{R_e} (.)\, dR_e$$



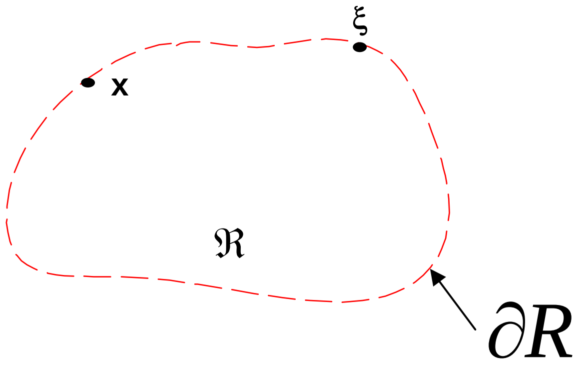
Bases y fundamentos:
Método de los elementos de contorno (BIEM)



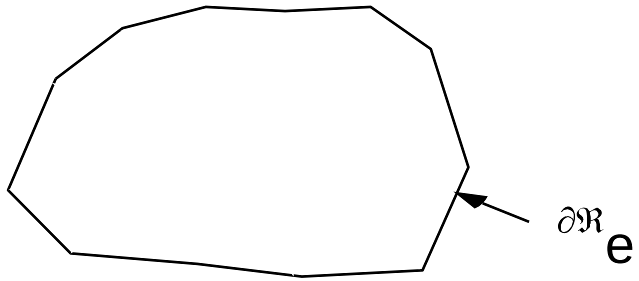
$$L[\phi] = \cancel{\chi(x,t)}$$

$$L^{*}[\psi] = -\delta(x,\xi)$$

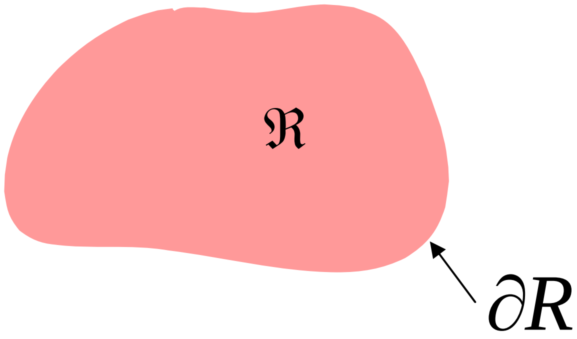
$$c(\theta)\phi(\xi) + \int_{\partial R}\left[h(x,\xi)\ \phi(x) - g(x,\xi)\frac{\partial\phi(x)}{\partial n}\right] d\Gamma_x = \cancel{\int_{R} \chi(x,t)\psi(x,\xi,t)\ dR_x}$$



$$c(\theta)\phi(\xi) + \int_{\partial R}\left[h(x,\xi)\ \phi(x) - g(x,\xi)\frac{\partial \phi(x)}{\partial n}\right] d\Gamma_x = 0$$



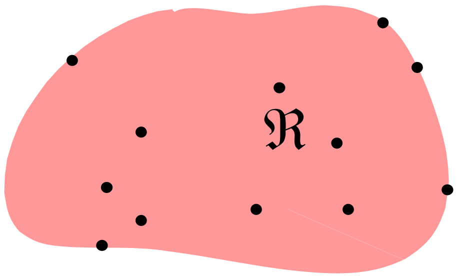
$$[C]\{\phi\} + [H]\{\phi\} - [G]\left\{\frac{\partial \phi}{\partial n}\right\} = 0$$



$$L[\phi] = \chi(x,t)$$

$$c(\theta)\phi(\xi) + \int_{\partial R}\left[h(x,\xi)\ \phi(x) - g(x,\xi)\frac{\partial\phi(x)}{\partial n}\right] d\Gamma_x = \boxed{\int_R \chi(x,t)\psi(x,\xi,t)\ dR_x}$$

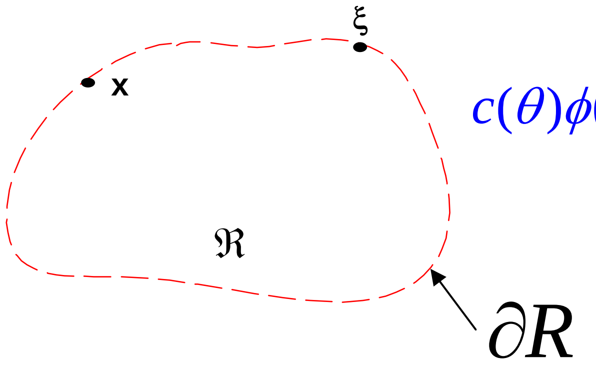
Método de Reciprocidad Dual (DRM)



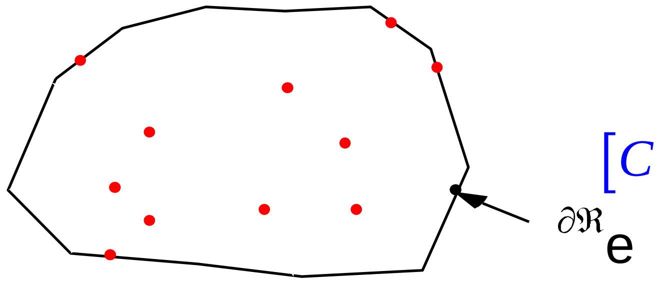
$$\chi(x,t) = \sum_{k=1}^{M} \alpha_k(t)\ f_k(x)$$

$$L[\zeta_k] = f_k(x)$$

$$\int_R \chi(x,t)\psi(x,\xi,t)\ dR_x \Rightarrow \left([C]\{\zeta\} + [H]\{\zeta\} - [G]\left\{\frac{\partial\zeta}{\partial n}\right\}\right)[F]^{-1}\{\chi\}_{RD} \equiv [\tau]\{\chi\}_{RD}$$

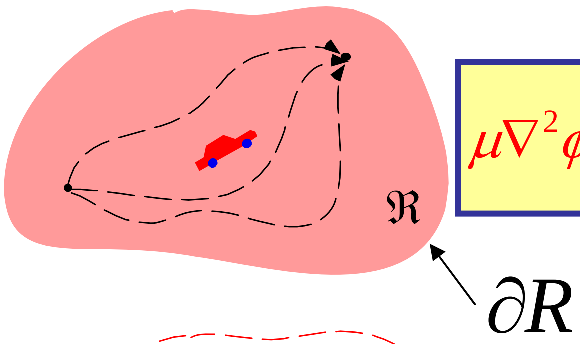


$$c(\theta)\phi(\xi) + \int_{\partial R}\left[h(x,\xi)\,\phi(x) - g(x,\xi)\frac{\partial\phi(x)}{\partial n}\right] d\Gamma_x = \boxed{\int_R \chi(x,t)\psi(x,\xi,t)\,dR_x}$$

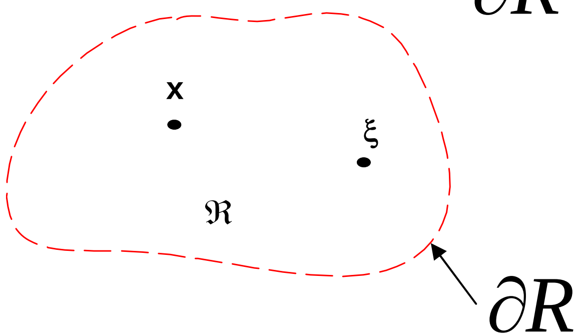


$$[C]\{\phi\} + [H]\{\phi\} - [G]\left\{\frac{\partial\phi}{\partial n}\right\} = \boxed{[\tau]\{\chi\}_{RD}}$$

Bases y fundamentos:
Flujo de tráfico bi-dimensional y uni-bien



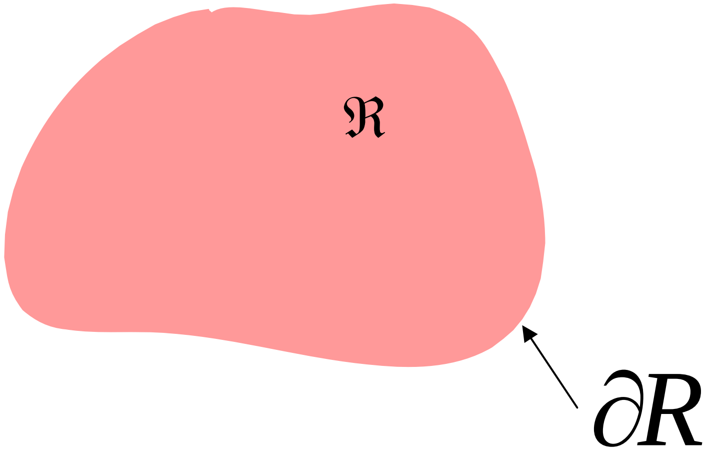
$$\mu\nabla^2\phi(x,t) - \bar{u}(t)\nabla\phi(x,t) = \frac{\partial\phi(x,t)}{\partial t} + \nabla[\tilde{u}(x,t)\phi(x,t)] - \rho(x,t)$$



$$\mu\nabla^2\psi(x,\xi,t) + \bar{u}(t)\cdot\nabla\psi(x,\xi,t) = -\delta(\xi)$$

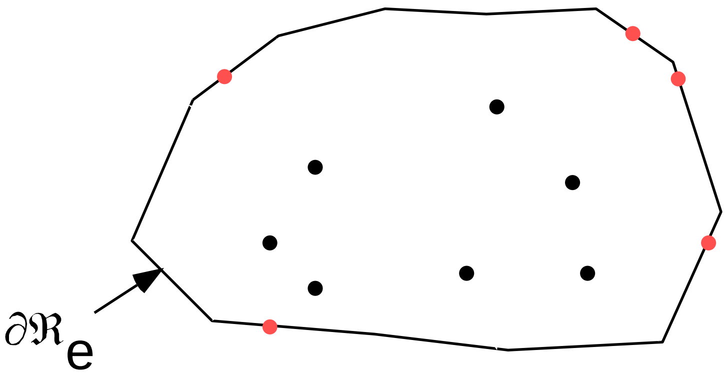
$$\begin{aligned}
&\phi(x,t) + \mu\int_{\partial R}\left[\phi(x,t)\frac{\partial\psi(x,\xi,t)}{\partial n} - \psi(x,\xi,t)\frac{\partial\phi(x,t)}{\partial n}\right] d\Gamma_x \\
&\qquad + \int_{\partial R}\phi(x,t)\psi(x,\xi,t)\bar{u}_n(t)\; d\Gamma_x \\
&\qquad = -\int_R\left\{\frac{\partial\phi(x,t)}{\partial t} + \nabla\cdot[\tilde{u}(x,t)\phi(x,t)] - \rho(x,t)\right\}\psi(x,\xi,t)\; dR_x
\end{aligned}$$

Bases y fundamentos:
Flujo de tráfico bi-dimensional y uni-bien



$$\int_{\partial R} (.)\, d\Gamma_x \qquad \int_{R} (.)\, dR_x$$

$$\int_{R} (.)\, dR_x \longrightarrow \int_{\partial R} (.)\, d\Gamma_x$$



$$\int_{\partial R} (.)\, d\Gamma_x \longrightarrow \sum_e \int_{\partial R_e} (.)\, d\Gamma_e \qquad \int_{\partial R} (.)\, d\Gamma_x \longrightarrow \sum_e \int_{\partial R_e} (.)\, d\Gamma_e$$

Término derivada en tiempo

$$\dot{\phi}(x,t)=\frac{\partial\phi(x,t)}{\partial t}=\sum_{k=1}^{K_1} f_k(x)\cdot\alpha_k(t)$$

Término distribución espacial

$$\rho(x,t)=\sum_{k=1}^{K_2} s_k(x)\cdot\beta_k(t)$$

Término de divergencia

$$\nabla\cdot\left[\tilde{u}(x,t)\phi(x,t)\right]=\sum_{k=1}^{K_3} r_k(x)\cdot\gamma_k(t)$$

$$\left.\begin{array}{l} f_k(x)=f(\xi,x_k,\overline{u}(t)) \\ r_k(x)=r(\xi,x_k,\overline{u}(t)) \\ s_k(x)=s(\xi,x_k,\overline{u}(t)) \end{array}\right\}=\mu(4+9r+16r^2)-\left[\overline{u}(\xi_x-x_k)+\overline{v}(\xi_y-y_k)\right](2+3r+4r^2)$$

Ecuación de la Densidad de tráfico

$$[C]\{\phi\} + [H]\{\phi\} - [G]\left\{\frac{\partial \phi}{\partial n}\right\} = [\tau]\left(\left\{\frac{\partial \phi}{\partial t}\right\} - \{\rho\} + \left\{\nabla \cdot \left[\widetilde{u}\,\phi\right]\right\}\right)_{RD}$$

$$\nabla \cdot \left[\widetilde{u}\,\phi\right] = \nabla \cdot \left[\tilde{u}\right] \cdot \phi + \widetilde{u} \cdot \nabla\left[\phi\right]$$

Ecuación del Gradiente de la Densidad de tráfico

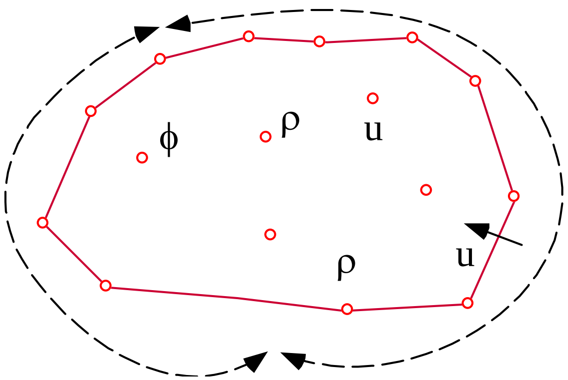
$$[A]\{\phi\} + [D]\{\nabla\phi\} + \left[\overline{H}\right]\{\phi\} - \left[\overline{G}\right]\left\{\frac{\partial \phi}{\partial n}\right\} = \left[\overline{\tau}\right]\left(\left\{\frac{\partial \phi}{\partial t}\right\} - \{\rho\} + \left\{\nabla \cdot \left[\widetilde{u}\,\phi\right]\right\}\right)_{RD}$$

Densidad

$$[C]\{\phi\} + [H]\{\phi\} - [G]\left\{\frac{\partial \phi}{\partial n}\right\} = [\tau]\left(\left\{\frac{\partial \phi}{\partial t}\right\} - \{\rho\} + \left\{\nabla \cdot \left[\tilde{u}\,\phi\right]\right\}\right)_{RD}$$

Gradiente

$$[A]\{\phi\} + [D]\{\nabla \phi\} + \left[\overline{H}\right]\{\phi\} - \left[\overline{G}\right]\left\{\frac{\partial \phi}{\partial n}\right\} = \left[\overline{\tau}\right]\left(\left\{\frac{\partial \phi}{\partial t}\right\} - \{\rho\} + \left\{\nabla \cdot \left[\tilde{u}\,\phi\right]\right\}\right)_{RI}$$



$$\frac{\partial \phi(\xi,t)}{\partial n} \qquad \phi\,(\xi,t)$$

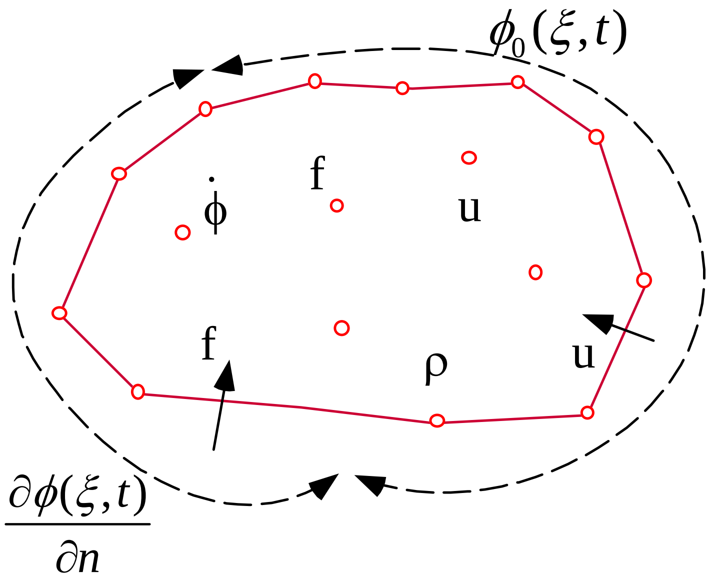
Discretización espacial

$$\left\{\frac{\partial \phi(x,t)}{\partial t}\right\} = \frac{\{\phi\}^{m+1} - \{\phi\}^{m}}{\Delta t}$$

Discretización temporal

$$\left[\hat{H}\right]\cdot\{\phi\}^{m+1} - \left[\hat{G}\right]\cdot\{\mathbf{q}\}^{m+1} = \left\{\hat{d}\right\}^{m}$$

Implementación numérica y Algoritmo



DEFINICION DE LA GEOMETRIA

- Discretización del Contorno
- Puntos internos
- Parámetros temporales
  - ➢ integración: θ_ϕ, θ_q
  - ➢ paso: Δt
  - ➢ intervalo: (t_0, t_∞)

$$t^m = t_0 + m\ \Delta t$$

CONDICIONES DE CONTORNO

- Densidad y Fluxes: $\{\phi\}^m, \left\{\dfrac{\partial\phi}{\partial n}\right\}^m$
- Velocidades: $\{u\}^m$
- Generación y Atracción de viajes: $\{\rho\}^m$

CONDICIONES INTERNAS

- Densidades: $\{\phi\}^m$
- Velocidades: $\{u\}^m$
- Generación y Atracción de viajes: $\{\rho\}^m$

Caso 1

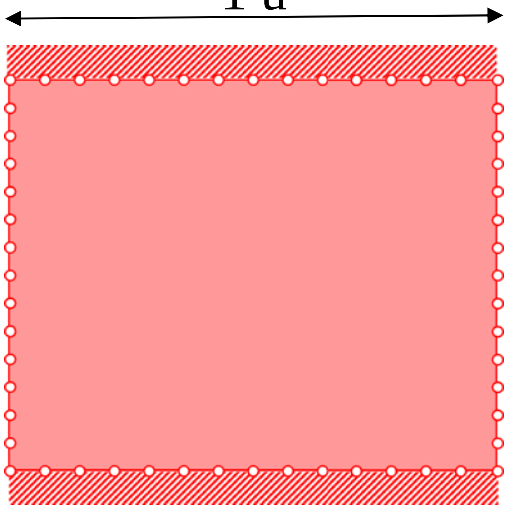
Caso estacionario

$$\mu\nabla^2\phi(x,t) - \nabla\cdot(u\phi) = 0$$

$$\frac{\partial\phi}{\partial n} = 0$$

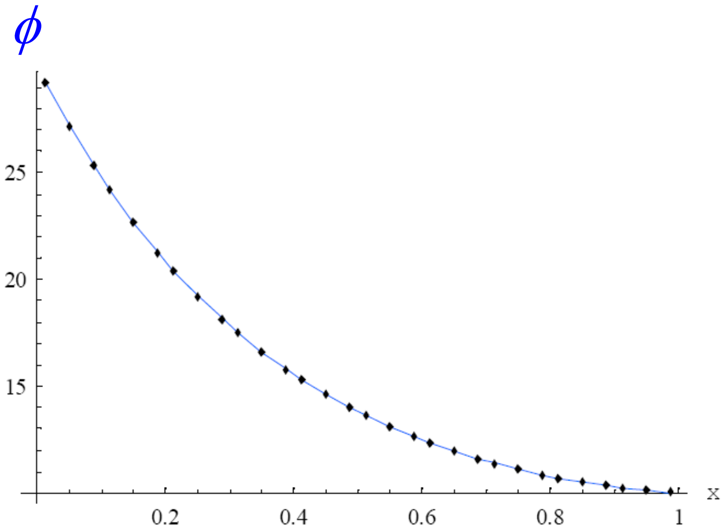
1 u

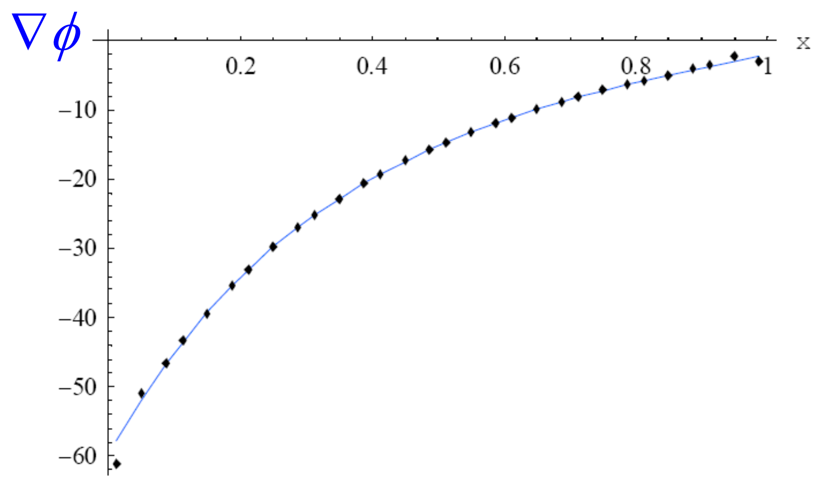
$$\phi_a = 30$$



$$\phi_b = 10$$

$$\frac{\partial\phi}{\partial n} = 0$$





Caso 2

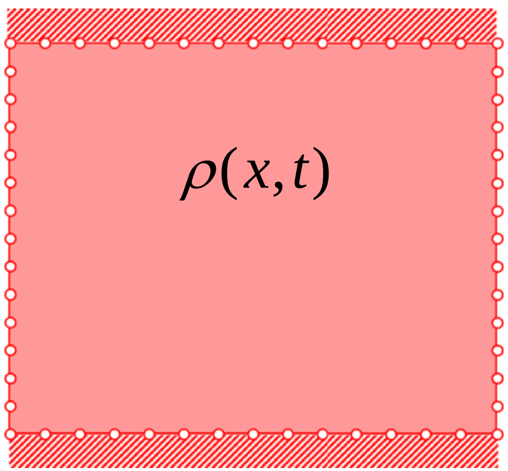
Caso transitorio

$$\mu\nabla^2\phi(x,t) - \nabla\cdot(u(x,t)\ \phi(x,t)) = \frac{\partial\phi(x,t)}{\partial t} - \rho(x,t)$$

$$\frac{\partial\phi}{\partial n} = 0$$

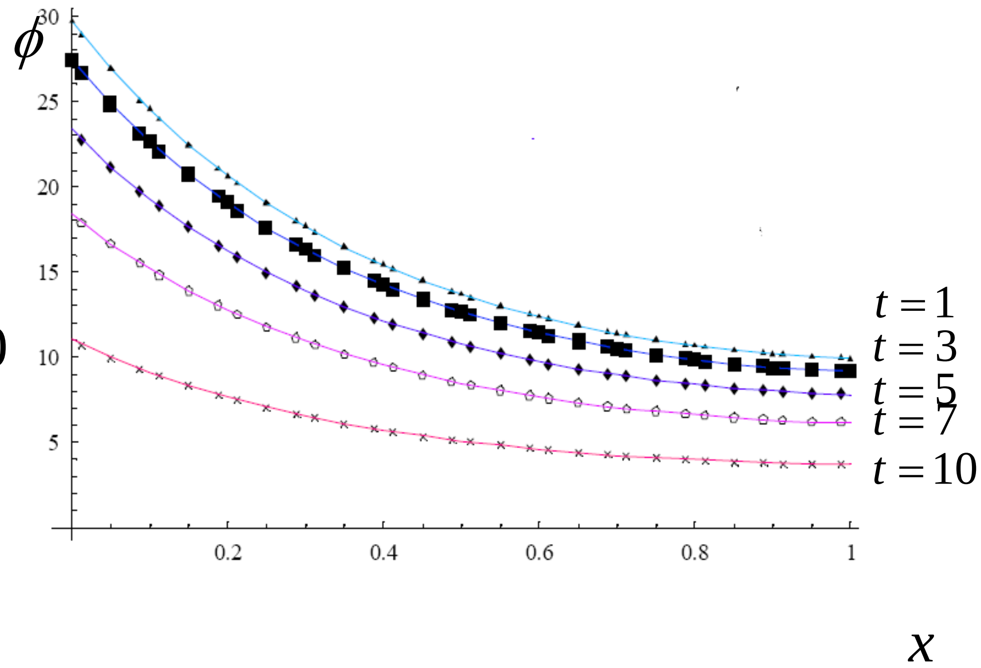
1 u

$\phi_a = 30$

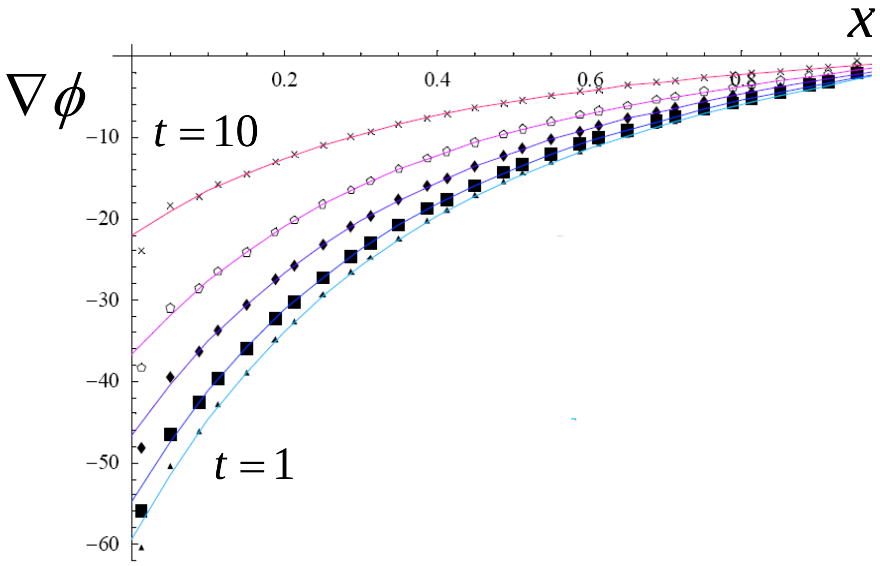


$\phi_b = 10$

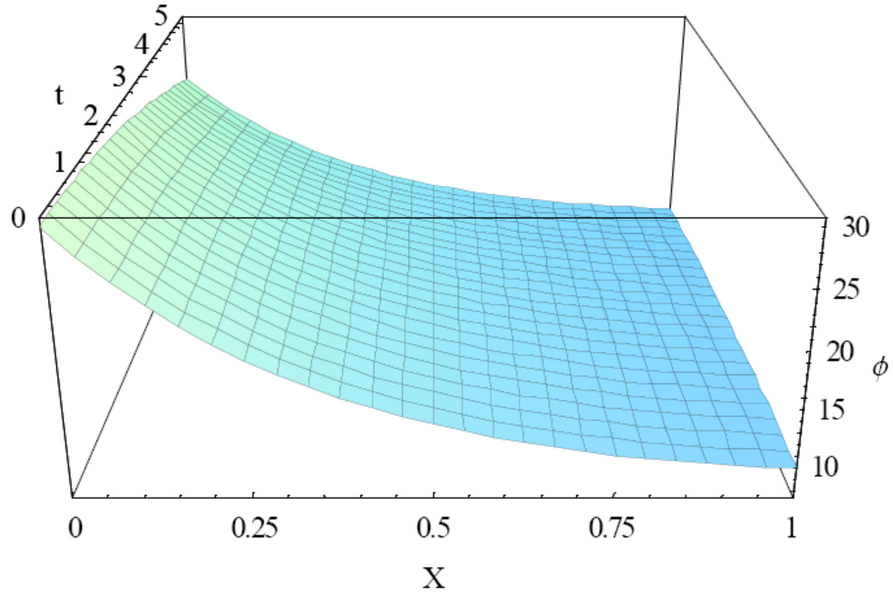
$$\frac{\partial\phi}{\partial n} = 0$$



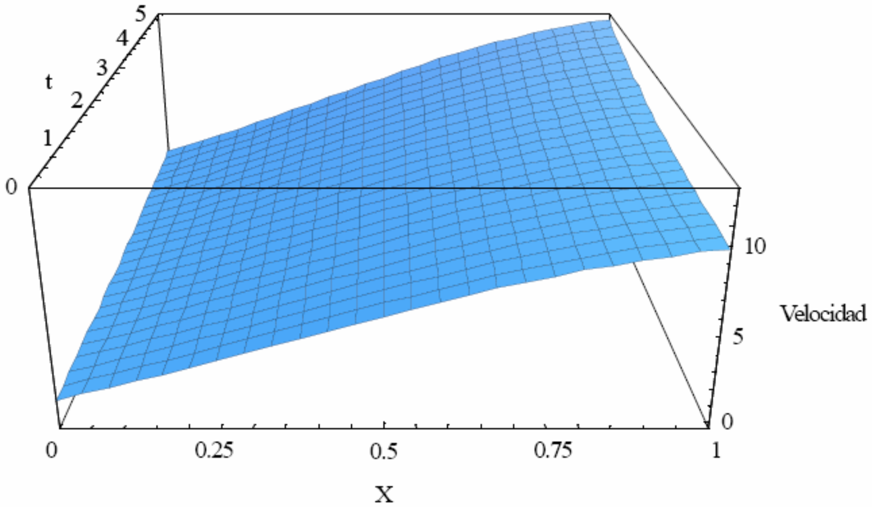
Caso 2



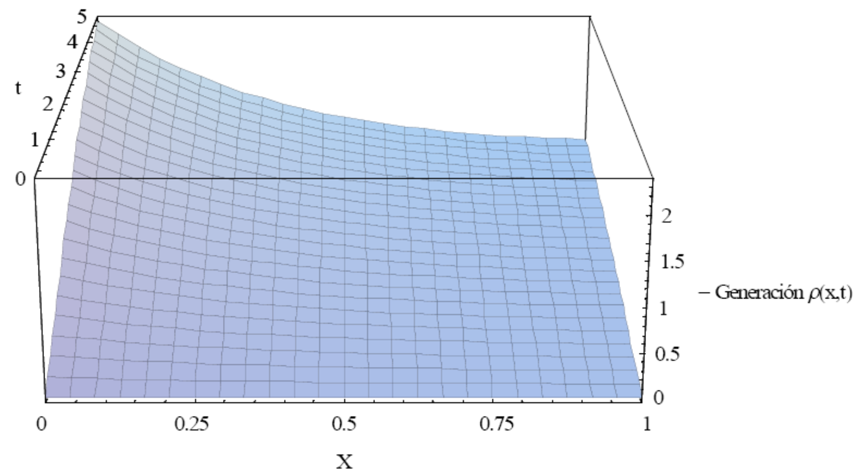
$\nabla\phi$
x
0.2
0.4
0.6
0.8
−10
−20
−30
−40
−50
−60
$t=10$
$t=1$



t
0
1
2
3
4
5
30
25
20
15
10
ϕ
0
0.25
0.5
0.75
1
X



t
0
1
2
3
4
5
10
5
0
Velocidad
0
0.25
0.5
0.75
1
x

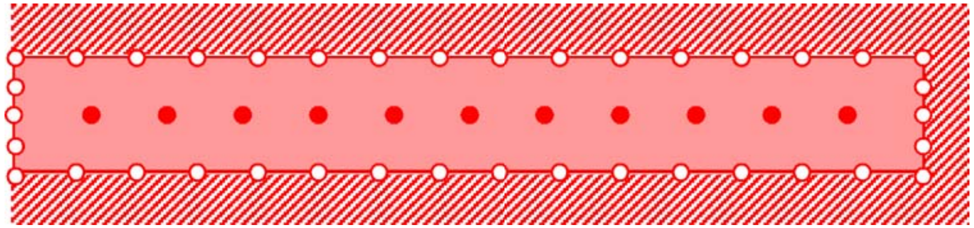


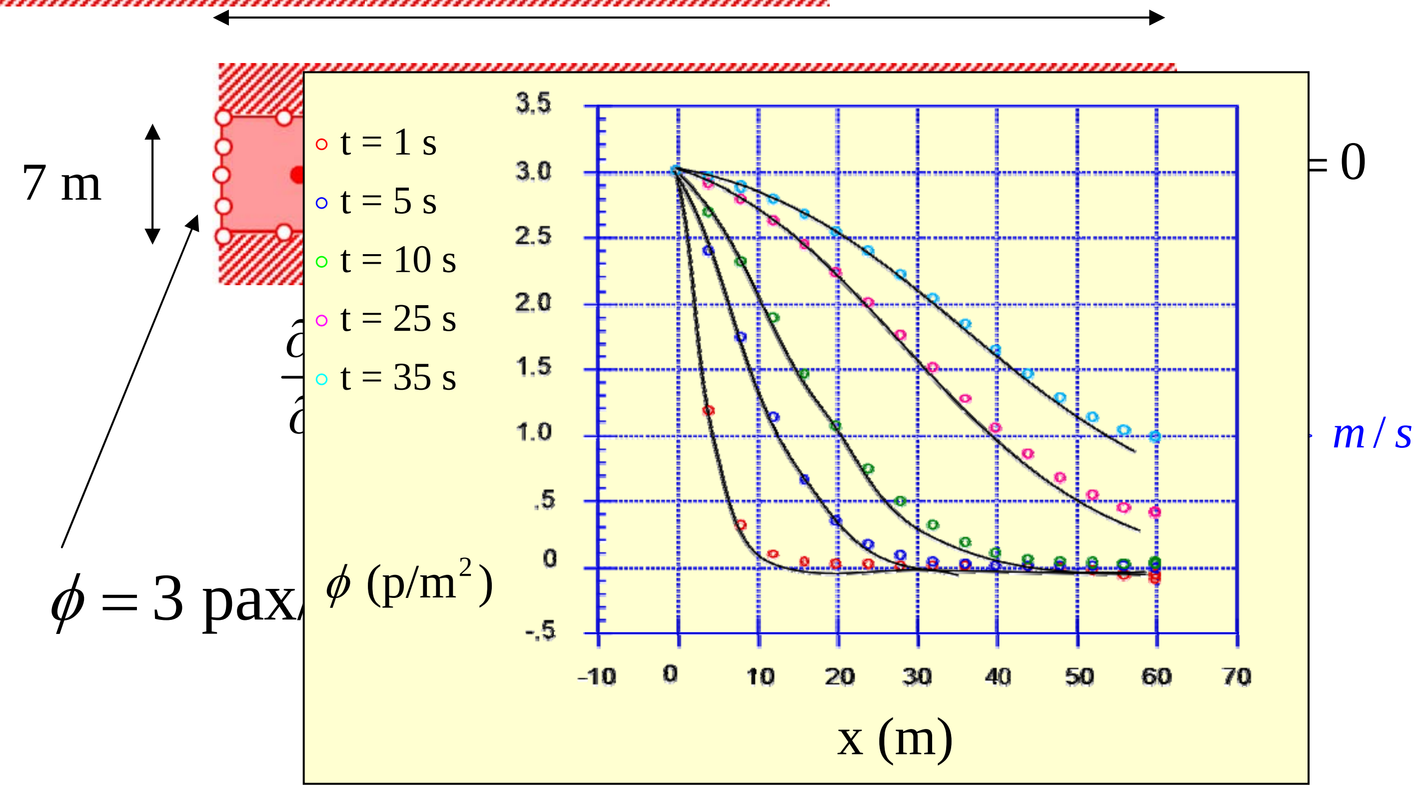
t
0
1
2
3
4
5
2
1.5
1
0.5
0
– Generación ρ(x,t)
0
0.25
0.5
0.75
1
x

Caso 3

Caso transitorio

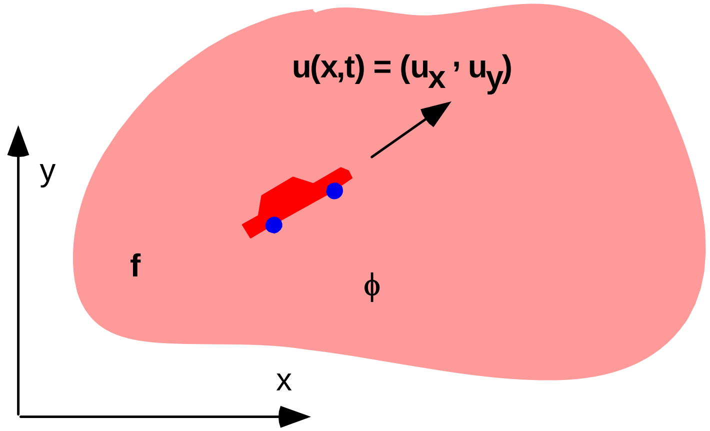
$$\mu\nabla^2\phi(x,t) - \nabla\cdot(u(x,t)\ \phi(x,t)) = \frac{\partial\phi(x,t)}{\partial t}$$





Flujo de tráfico bi-dimensional y multi-bien

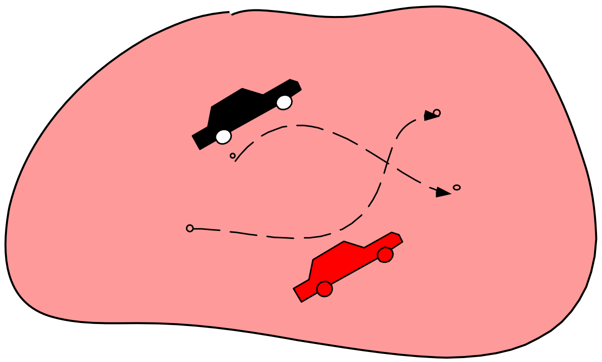
Flujo de tráfico uni-bien



$$\frac{\partial \phi(x,t)}{\partial t} + \nabla \cdot \mathbf{f}(\mathbf{x},t) = \rho(\mathbf{x},t)$$

$$\mathbf{f}(\mathbf{x},t) = \underbrace{\phi(x,t)\; u(x,t)}_{convectiva} \underbrace{-\mu\; \nabla \phi(x,t)}_{difusiva}$$

Flujo de tráfico multi-bien

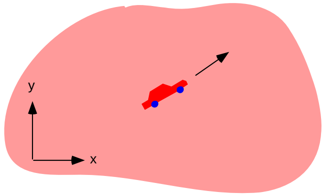


$$\frac{\partial \phi^b(x,t)}{\partial t} + \nabla \cdot \mathbf{f}^b(\mathbf{x},t) = \rho^b(\mathbf{x},t) \qquad \forall b = 1,\dots, NB$$

$$\mathbf{f}^b(\mathbf{x},t) = \underbrace{\phi^b(x,t)\; u^b(x,t)}_{convectiva} \underbrace{-\sum_{c=1}^{NB} \mu^c\; \nabla \phi^c(x,t)}_{difusiva}$$

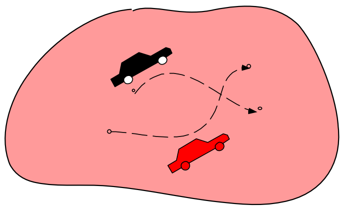
Flujo de tráfico uni-bien

Ecuación de advección-difusión



$$\mu\nabla^2\phi(x,t)-\overline{u}(t)\nabla\phi(x,t)=\frac{\partial\phi(x,t)}{\partial t}+\nabla[\tilde{u}(x,t)\phi(x,t)]-\rho(x,t)$$

Flujo de tráfico multi-bien



$$\mu^b\nabla^2\phi^b(x,t)-\overline{u^b}(t)\nabla\phi^b(x,t)+\sum_{c\neq b=1}^{NB}\mu^c\nabla^2\phi^c(x,t)=$$
$$\frac{\partial\phi^b(x,t)}{\partial t}+\nabla[\widetilde{u^b}(x,t)\phi^b(x,t)]-\rho^b(x,t)$$

Líneas investigadas:

Flujo de tráfico bi-dimensional y multi-bien

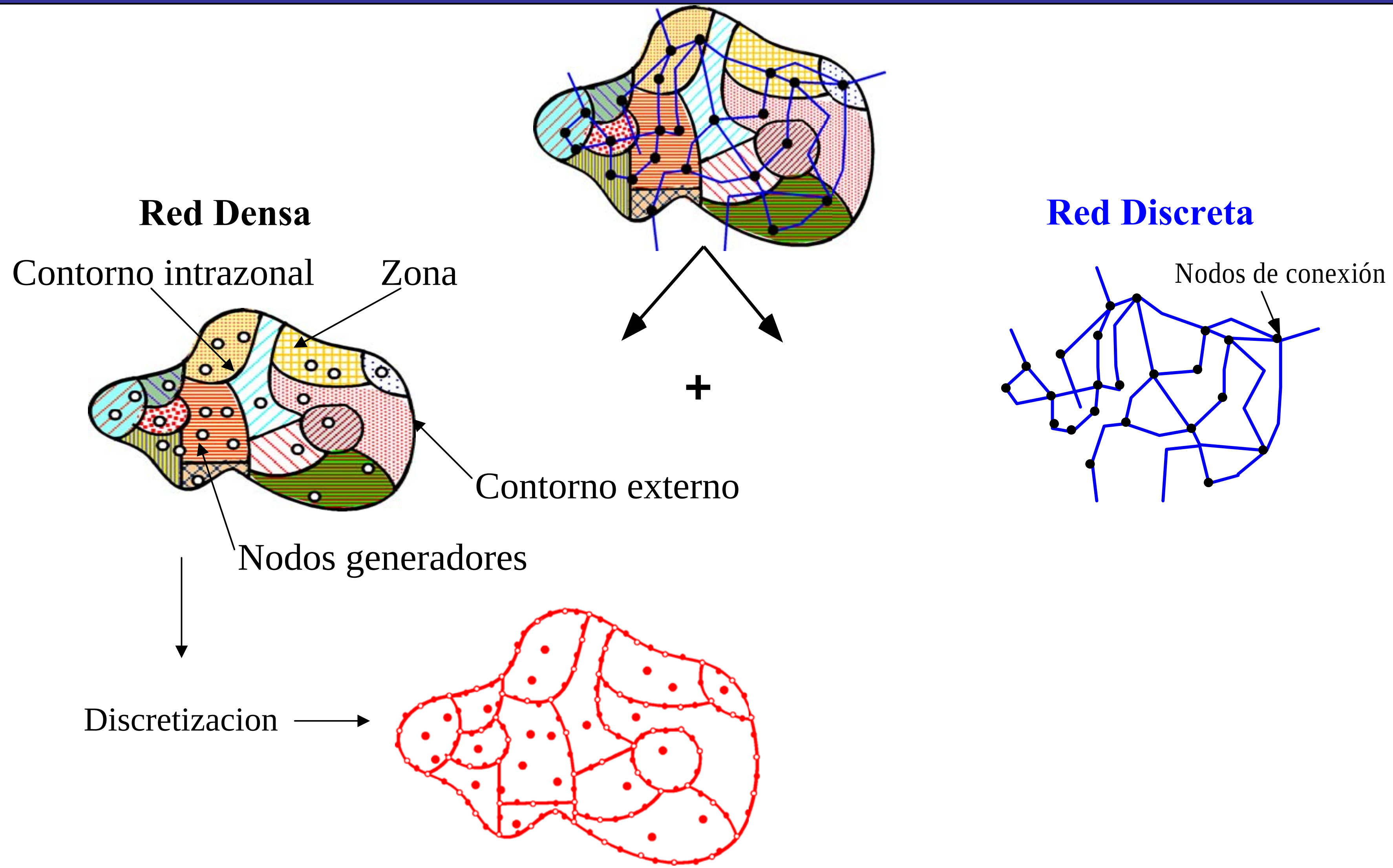
$$\sum_{c=1}^{NB}\left(\frac{\mu^c}{\mu^b}\left(\left\{h^b\right\}^T\cdot\left\{\phi^c\right\}-\left\{g^b\right\}^T\cdot\left\{\frac{\partial\phi^c}{\partial\mathbf{n}}\right\}\right)+\right.$$
$$\left.+\left(1-\delta_{bc}\right)\left(\left\{h^b\right\}^T\cdot[\varsigma]-\left\{g^b\right\}^T\cdot\left[\frac{\partial\varsigma}{\partial\mathbf{n}}\right]\right)[F^b]^{-1}\frac{\mu^c}{\mu^b}\left\{\bar{\mathbf{u}}^b\cdot\nabla\phi^c\right\}\right)=$$
$$=\left(\left\{h^b\right\}^T\cdot[\varsigma]-\left\{g^b\right\}^T\cdot\left[\frac{\partial\varsigma}{\partial\mathbf{n}}\right]\right)[F^b]^{-1}\cdot$$
$$\cdot\left\{\frac{\partial\phi^b(\mathbf{x},t)}{\partial t}-\boldsymbol{\rho}^b(\mathbf{x},t)+\nabla\cdot\left[\tilde{\mathbf{u}}^b(\mathbf{x},t)\phi^b(\mathbf{x},t)\right]\right\}$$

Ecuación de la densidad del bien *i*

Ecuación del gradiente de densidad del bien *i*

$$\sum_{c=1}^{NB}\left(\frac{\mu^c}{\mu^b}\left(\mathbf{d}_{\xi_{x_i}}(\theta)\nabla\phi^c(\boldsymbol{\xi})+\left\{h^b_{\xi_{x_i}}(\boldsymbol{\xi})\right\}^T\left\{\phi^c\right\}-\left\{g^b_{\xi_{x_i}}(\boldsymbol{\xi})\right\}^T\left[\frac{\partial\phi^c}{\partial\mathbf{n}}\right]\right)+\right.$$
$$+\left(1-\delta_{bc}\right)\frac{\mu^c}{\mu^b}\bar{\mathbf{u}}^b\cdot\left(\mathbf{d}_{\xi_{x_i}}(\theta)\nabla\zeta_k(\boldsymbol{\xi})+\left\{h^b_{\xi_{x_i}}(\boldsymbol{\xi})\right\}^T[\zeta]-\left\{g^b_{\xi_{x_i}}(\boldsymbol{\xi})\right\}^T\left[\frac{\partial\zeta}{\partial\mathbf{n}}\right]\right)\cdot$$
$$\left.\cdot\left[F^b\right]^{-1}\left\{\nabla\phi^c\right\}\right)=$$
$$=\left(a_{\xi_{x_i}}(\theta)\zeta_k(\boldsymbol{\xi})+\mathbf{d}_{\xi_{x_i}}(\theta)\nabla\zeta_k(\boldsymbol{\xi})+\left\{h^b_{\xi_{x_i}}(\boldsymbol{\xi})\right\}^T[\zeta]-\left\{g^b_{\xi_{x_i}}(\boldsymbol{\xi})\right\}^T\left[\frac{\partial\zeta}{\partial\mathbf{n}}\right]\right)$$
$$\cdot\left[F^b\right]^{-1}\left\{\frac{\partial\phi^b(\mathbf{x},t)}{\partial t}-\boldsymbol{\rho}^b(\mathbf{x},t)+\nabla\cdot\left[\tilde{\mathbf{u}}^b(\mathbf{x},t)\phi^b(\mathbf{x},t)\right]\right\}$$

Líneas investigadas:
Modelización mixta continuo-discreta



Conclusiones

Conclusión Principal

Metodología y Formulación para el análisis del flujo de tráfico en redes densas

VENTAJAS

- Modelado sencillo y realista
  - o Pocos datos
  - o Modelización económica
- Facilidad de introducir modificaciones
  - o actualizaciones
  - o intervenciones
- Sistemas informáticos no especializados
- Personal no especializado

DESVENTAJAS

- Está poco desarrollado
- Robusto ?
- Contrastación con la realidad ?.

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